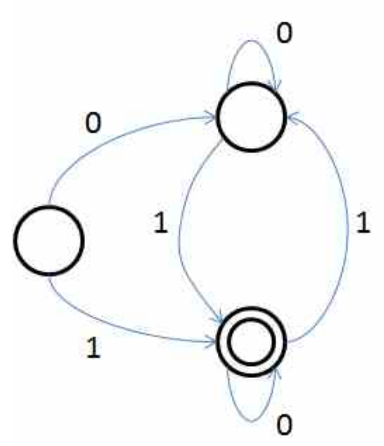
CPS 102: Discrete Mathematics	**Instructor:** Bruce Maggs
Quiz 1	**Date:** Monday October 4, 2010

NAME:

Prob #.	Score	Max Score
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total		100

Problem 1
Use a regular expression to describe the language accepted by the following deterministic finite automata (DFA).



Problem 2
Draw a DFA that accepts the language $\varepsilon + (aab^*a) + (bba^*)$.

Problem 3
Use a regular expression to describe the set of strings over the alphabet $\{0, 1\}$ in which every 1 is immediately followed by a zero.

Problem 4

Draw a DFA that accepts the set of strings of 0's and 1's that contain at least one instance of three consecutive 0's.

Problem 5

Prove that the set $\{01, 01001, 010010001, 01001000100001, \ldots\}$ cannot be accepted by any DFA.

Problem 6
Show that the number of different languages over the alphabet $\Sigma = \{0, 1\}$ that are accepted by deterministic finite automata with only two states is finite.

Problem 7

Prove by contradiction: There are infinitely many even numbers.

Problem 8

A *rational* number is a real number that can be expressed as the ratio of two integers. An *irrational* number is a real number that is not rational. Provide an indirect proof of the following statement, i.e., prove the contrapositive. If a and b are real numbers and $a \cdot b$ is an irrational number, then either a or b is irrational.

Problem 9

Prove by induction that

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+\cdots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1}.$$

Problem 10

Suppose that there are only two types of postage stamps, 4-cent stamps and 5-cent stamps. Prove that any amount of postage of 12 cents or greater can be made up out of 4-cent and 5-cent stamps. Hint: It is possible to prove this using strong induction over N.