CPS 102: Discrete Mathematics **Instructor:** Bruce Maggs

Quiz 2 **Date:** Wednesday November 10, 2010

NAME:

Prob #	Score	Max Score
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total		100

Problem 1

A crafty professor decides to test his students' knowledge of the principle of inclusion and exclusion, so he sets up the following problem. "Seven students in the class like Hot Topic, nine students like either Hot Topic or Hello Kitty, and fifteen like Hello Kitty. How many like both Hot Topic and Hello Kitty?" The professor thinks that the answer is thirteen. Why does he think this, and what's wrong with this problem?

Problem 2

Use induction to prove that $2n + 1 \leq 2^n$ if n is an integer greater than 2.

Problem 3

Use induction to prove that for all integers $n \geq 1$,

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = (1 + 2 + 3 + \cdots + n)^2.$$

Problem 4

How many bit strings of length 14 contain

a) exactly four 1s?

b) at most four 1s?

Problem 5

Mr. Jones has two children, at least one of which is a girl. What is the probability that both children are girls? (Assume that at birth any child is equally likely to be a boy or a girl, independent of any other child.) Hint: Let A denote the event that both children are girls, and let B denote the event that at least one child is a girl. What are we asking in terms of A and B?

Problem 6

Suppose that more than half of the numbers in the set $\{1, 2, \ldots, 2n\}$ are selected. Use the pigeonhole principle to prove that some two of the selected numbers have the property that one divides the other. Hint: $1|2$, $2|4$, $3|6$, etc.

Problem 7

Find a closed-form solution to the following continued fraction

$$X = 1 + \cfrac{2}{1 + \cfrac{2}{1 + \cfrac{2}{1 + \cfrac{2}{1 + \dots}}}}.$$

Problem 8

Before every class, the professor attempts to guess the name of each of the twenty-three students. Suppose that for each student, the professor succeeds with probability $1/7$, independent of whether he succeeds for any other students (and, sadly, independent of whether he has succeeded on any previous day). What is the probability that the professor correctly guesses the names of at least three students?

Problem 9

Suppose that Alice wishes to send two messages m_1 and m_2 to Bob using the RSA cryptosystem. Alice sends m_1^e and $m_2^e \pmod{n}$ to Bob, where e is Bob's public key. Now suppose that there is an eavesdropper (traditionally named "Eve") that sees everything sent from Alice to Bob. One potential vulnerability with RSA is that even without knowing m_1 and m_2, Eve can tell if Alice later encrypts and sends $m_1 \cdot m_2$ to Bob. Explain how Eve can do this.

Problem 10

What is the probability that a five-card poker hand contains a straight, that is, five cards that have consecutive ranks, assuming that an ace can either be used as a card with rank one (the lowest rank) or a card with rank thirteen (the highest rank).