Due Date: Noon, December 5, 2011

Problem 1 (20pts) Let P be a convex polygon with n vertices. The *weight* of a triangulation of P is the sum of the lengths of its diagonals. Describe an $O(n^3)$ time algorithm for computing the *minimum weight* triangulation of P. (**Hint:** *Use dynamic programming.*)

Problem 2 (20pts) Let M be a triangulation of the convex hull of a set V of n points in $\mathbb{R}^2$. Let $h : V \to \mathbb{R}$ be a height function on the points of S; assume that the height of each vertex is distinct. Using linear interpolation inside each triangle of M, we can define the height of any point in M, i.e., we have a function $h : M \to \mathbb{R}$. The graph of M is a triangulated piecewise-linear surface in $\mathbb{R}^3$. For a value $z \in \mathbb{R}$, the *level set* of M at z, denoted by M_z, is $M_z = \{x \in M \mid h(x) = z\}$. M_z is a collection polygonal cycles; each cycle is called a *contour* at height z. Describe an algorithm that can preprocesses M, in $O(n \log n)$ time, into a linear-size data structure so that for a any value z, the vertices of all the contours at height z can be computed in $O(\log n + k)$ time; here k is the number of vertices in the contour.

Problem 3 (30pts) Let S be a set of n points in $\mathbb{R}^2$. For a circle C, let $\omega(C, S)$ be the maximum distance between C and a point of S, i.e., if c and r are the center and radius of C, then $\omega(C, S) = \max_{p \in S} |\|p - c\| - r|$. Let $C^* = \arg\min_C \omega(C, S)$, where the minimum is taken over all circles in $\mathbb{R}^2$. Let $\mathrm{Vor}(S)$, $\mathrm{Vor}_f(S)$ be the nearest-neighbor and the farthest-neighbor Voronoi diagrams of S.

- Show that the center of C^* is a vertex of $\mathrm{Vor}(S)$, a vertex of $\mathrm{Vor}_f(S)$, or an intersection point of the edges of the two diagrams.
- Show that C^* can be computed in $O(n^2)$ time.

Problem 4 (30pts) Let $X, Y \subset \mathbb{R}^2$ be two sets. The *Minkowski sum* of X and Y is

$$X \oplus Y = \{x + y \mid x \in X, y \in Y\}.$$

Let P_1, P_2 be two convex n-gons in $\mathbb{R}^2$, and let S_1, S_2 be the set of the vertices in P_1 and P_2, respectively. Set $P = P_1 \oplus P_2$. Show that

- $P = \mathrm{conv}(S_1 \oplus S_2)$;
- each edge of P is of the form $e \oplus v$, where e is an edge P_1 and $v \in S_2$, or e is an edge of P_2 and $v \in S_1$;
- P can be computed in $O(n)$ time, assuming that S_1 and S_2 are given as sorted in clockwise direction.