Due Date: October 25, 2011

Problem 1: [10pts] Let $P = \langle p_0, \ldots, p_{n-1}\rangle$ and $Q = \langle q_0, \ldots, q_{n-1}\rangle$ be two nonintersecting convex polygons in $\mathbb{R}^2$. Show that the common tangent, both inner and outer, can be computed in $O(\log n)$ time. You can assume that the sequence of vertices of P (and Q) is stored in an array.

Problem 2: [15pts] Let P be a set of n points in $\mathbb{R}^2$. We define a map $N : \mathbb{S}^1 \to P$, where $N(u) = \arg\max_{p\in P}\langle p, u\rangle$. N induces a subdivision P^* of $\mathbb{S}^1$. How fast can P^* be computed? ($\langle p, u\rangle$ is the inner product of the two vectors.)

We call a pair of vertices $p, q \in P$ *antipodal* if there are two parallel lines h_p, h_q passing through p and q, respectively, so that P lies between them. The *width* of P is the minimum width of a strip that contains P. Show that a strip that realizes the width of P contains an edge of $CH(P)$ and an antipodal pair on its boundary. Describe an $O(n \log n)$ algorithm for computing the width of P. (**Hint:** *Use the function* N.)

Describe an $O(n \log n)$ algorithm for computing the minimum-area rectangle (of arbitrary orientation) that contains P.

Problem 3: [10pts] Let $\mathcal{D}$ be a set of n circular disks in $\mathbb{R}^2$. Show that the union of disks in $\mathcal{D}$ has $O(n)$ vertices and edges, and that it can be computed in $O(n \log n)$ randomized expected time.

Problem 4: [15pts] Let S be a set of n points in $\mathbb{R}^2$. For each point $p \in S$, define $\mathrm{Vor}_f(p) = \{x \in \mathbb{R}^2 \mid \|xp\| \geq \|xq\| \ \forall q \in S\}$ and the *farthest point Voronoi diagram* $\mathrm{Vor}_f(S) = \{\mathrm{Vor}_f(p) \mid p \in S\}$. Show that $\mathrm{Vor}_f(p)$ is nonempty and unbounded if and only if p is a vertex of the convex hull of S, and that the edges of $\mathrm{Vor}_f(S)$ form a tree.