## Due Date: November 14, 2011

Problem $1(10 p t s)$ Let $L$ be a set of $n$ lines in $\mathbb{R}^{2}$. For a cell $c$ in the arrangement $\mathcal{A}(L)$, let $|c|$ be the number of edges in $c$. Show that

$$
\sum_{c \in \mathcal{A}(L)}|c|^{2}=O\left(n^{2}\right)
$$

Problem 2 (10pts) Describe an $O\left(n^{3}\right)$ time algorithm to compute a minimum-weight triangulation of a convex $n$-gon. (Hint: Use dynamic programming.)

Problem 3 (15pts) Let $S$ be a set of $n$ segments in $\mathbb{R}^{2}$. Preprocess $S$ into a data structure of size $O\left(n^{2}\right)$ so that for a query line $\ell$, the number of segments of $S$ intersecting $\ell$ can be computed in $O(\log n)$ time. Show that the data structure can be constructed in $O\left(n^{2} \log n\right)$ time. (Hint: Use duality.)

Problem 4 (20pts) Let $S$ be a set of $n$ segments in $\mathbb{R}^{2}$, let $W$ be a vertical strip that contains all segments of $S$, and let $\chi(S)$ be the number of intersection points in $S$. Show that:
(i) If the endpoints of $S$ lie on the boundary of $W$, then $\chi(S)$ can be computed in $O(n \log n)$ time.
(ii) If $m$ of the segments in $S$ have their endpoints lying in the interior of $W$, then $\chi(S)$ can be computed in $O\left(\left(m^{2}+n\right) \log n\right)$ time. (Hint: Use the solution to Problem 3 )
(iii) Use (i) and (ii) to show that $\chi(S)$ can be computed in $O\left(n^{4 / 3} \log n\right)$ time. (Hint: Use the cutting theoream.)

