CPS 296.1

Brief introduction to linear and mixed integer programming

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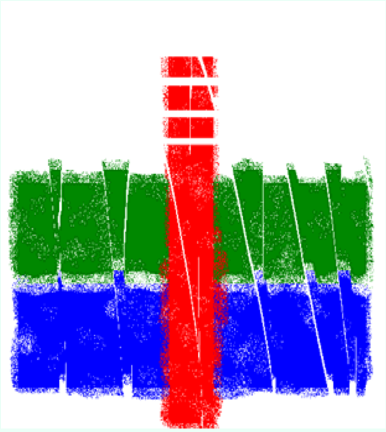
Linear programs: example

- We make reproductions of two paintings



- Painting 1 sells for \$30, painting 2 sells for \$20
- Painting 1 requires 4 units of blue, 1 green, 1 red
- Painting 2 requires 2 blue, 2 green, 1 red
- We have 16 units blue, 8 green, 5 red

$$\textit{maximize } 3x + 2y$$

subject to

$$4x + 2y \leqslant 16$$

$$x + 2y \leqslant 8$$

$$x + y \leqslant 5$$

$$x \geqslant 0$$

$$y \geqslant 0$$

Solving the linear program graphically

maximize $3x + 2y$

subject to

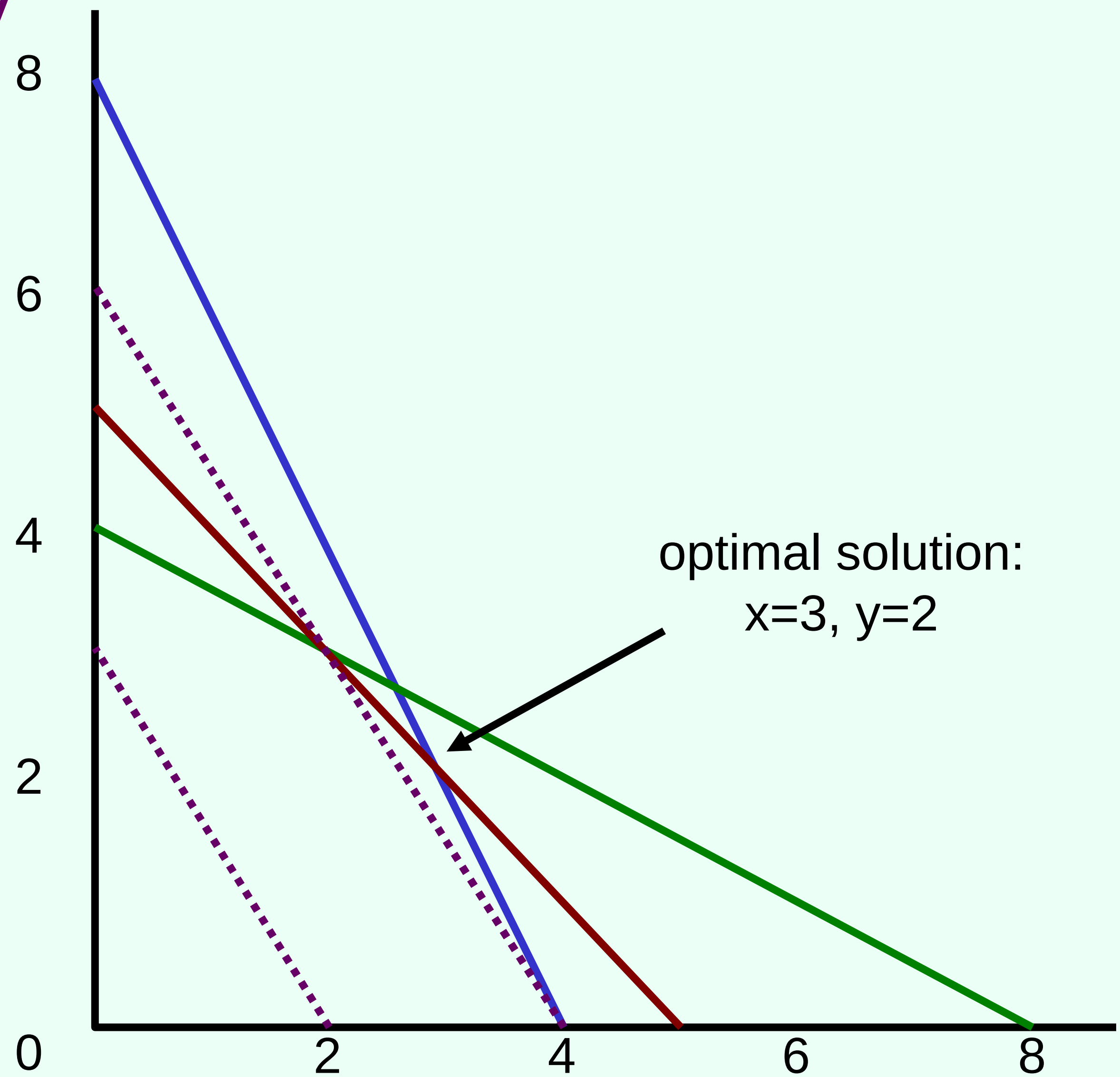
$4x + 2y \leqslant 16$

$x + 2y \leqslant 8$

$x + y \leqslant 5$

$x \geqslant 0$

$y \geqslant 0$



Modified LP

maximize $3x + 2y$

subject to

$4x + 2y \leqslant 15$

$x + 2y \leqslant 8$

$x + y \leqslant 5$

$x \geqslant 0$

$y \geqslant 0$

Optimal solution: $x = 2.5$, $y = 2.5$

Solution value = $7.5 + 5 = 12.5$

Half paintings?

Integer (linear) program

maximize $3x + 2y$

subject to

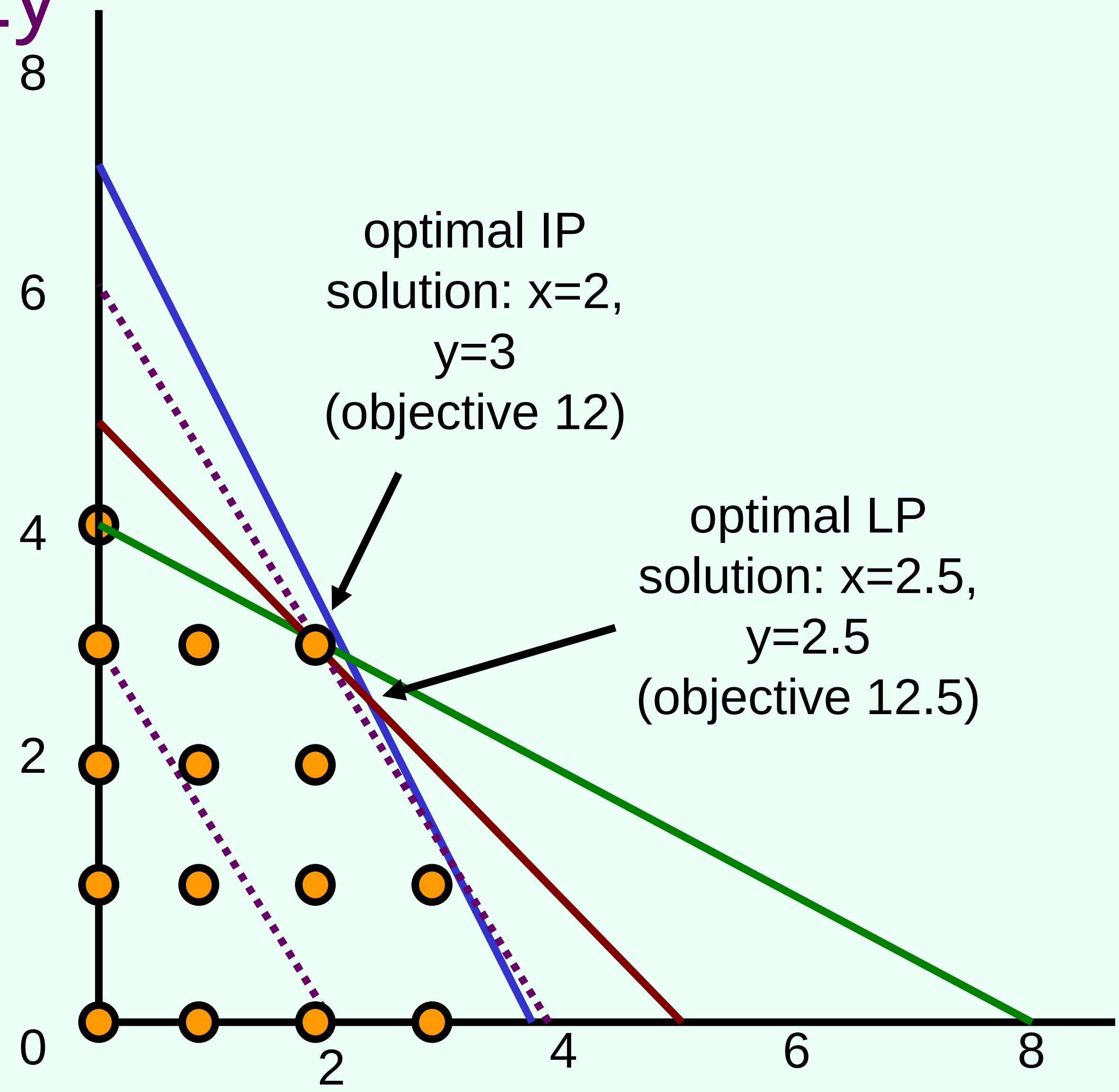
$4x + 2y \leqslant 15$

$x + 2y \leqslant 8$

$x + y \leqslant 5$

$x \geqslant 0$, integer

$y \geqslant 0$, integer



Mixed integer (linear) program

maximize $3x + 2y$

subject to

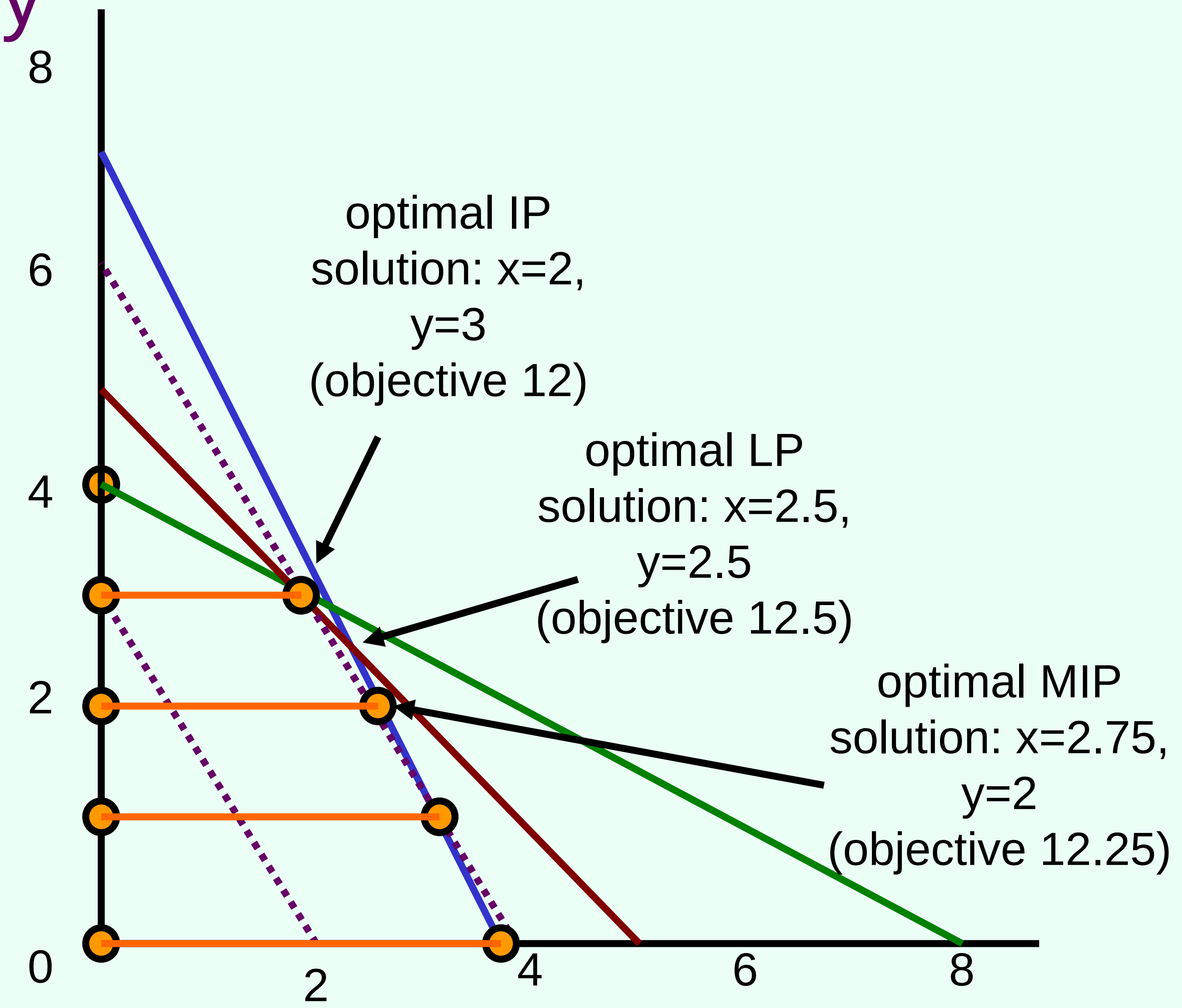
$4x + 2y \leqslant 15$

$x + 2y \leqslant 8$

$x + y \leqslant 5$

$x \geqslant 0$

$y \geqslant 0$, integer



Solving linear/integer programs

- Linear programs can be solved efficiently
  - Simplex, ellipsoid, interior point methods…
- (Mixed) integer programs are NP-hard to solve
  - Quite easy to model many standard NP-complete problems as integer programs (try it!)
  - Search type algorithms such as branch and bound
- Standard packages for solving these
  - GNU Linear Programming Kit, CPLEX, …
- LP relaxation of (M)IP: remove integrality constraints
  - Gives upper bound on MIP (~admissible heuristic)

Exercise in modeling: knapsack-type problem

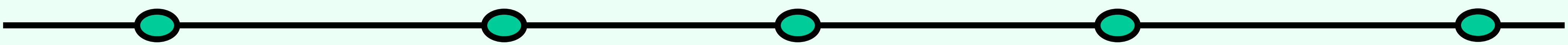
- We arrive in a room full of precious objects
- Can carry only 30kg out of the room
- Can carry only 20 liters out of the room
- Want to maximize our total value
- Unit of object A: 16kg, 3 liters, sells for $11
  - There are 3 units available
- Unit of object B: 4kg, 4 liters, sells for $4
  - There are 4 units available
- Unit of object C: 6kg, 3 liters, sells for $9
  - Only 1 unit available
- What should we take?

Exercise in modeling: cell phones (set cover)

- We want to have a working phone in every continent (besides Antarctica)
- … but we want to have as few phones as possible
- Phone A works in NA, SA, Af
- Phone B works in E, Af, As
- Phone C works in NA, Au, E
- Phone D works in SA, As, E
- Phone E works in Af, As, Au
- Phone F works in NA, E

Exercise in modeling: hot-dog stands

- We have two hot-dog stands to be placed in somewhere along the beach
- We know where the people that like hot dogs are, how far they are willing to walk
- Where do we put our stands to maximize #hot dogs sold? (price is fixed)



location: 1
#customers: 2
willing to walk: 4

location: 4
#customers: 1
willing to walk: 2

location: 7
#customers: 3
willing to walk: 3

location: 9
#customers: 4
willing to walk: 3

location: 15
#customers: 3
willing to walk: 2