CPS 296.1
Utility theory

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Risk attitudes

• Which would you prefer?
  – A lottery ticket that pays out $10 with probability 0.5 and $0 otherwise, or
  – A lottery ticket that pays out $3 with probability 1

• How about:
  – A lottery ticket that pays out $100,000,000 with probability 0.5 and $0 otherwise, or
  – A lottery ticket that pays out $30,000,000 with probability 1

• Usually, people do not simply go by expected value

• An agent is risk-neutral if she only cares about the expected value of the lottery ticket

• An agent is risk-averse if she always prefers the expected value of the lottery ticket to the lottery ticket
  – Most people are like this

• An agent is risk-seeking if she always prefers the lottery ticket to the expected value of the lottery ticket
Decreasing marginal utility

- Typically, at some point, having an extra dollar does not make people much happier (decreasing marginal utility)

![Graph showing utility and money]
• Lottery 1: get $1500 with probability 1
  – gives expected utility 2
• Lottery 2: get $5000 with probability .4, $200 otherwise
  – gives expected utility .4*3 + .6*1 = 1.8
  – (expected amount of money = .4*$5000 + .6*$200 = $2120 > $1500)
• So: maximizing expected expected utility is consistent with risk aversion
Different possible risk attitudes under expected utility maximization

- **Green** has decreasing marginal utility → risk-averse
- **Blue** has constant marginal utility → risk-neutral
- **Red** has increasing marginal utility → risk-seeking
- **Grey’s** marginal utility is sometimes increasing, sometimes decreasing → neither risk-averse (everywhere) nor risk-seeking (everywhere)
What is utility, anyway?

- Function \( u: O \rightarrow \mathbb{R} \) (\( O \) is the set of “outcomes” that lotteries randomize over)

- What are its units?
  - It doesn’t really matter
  - If you replace your utility function by \( u'(o) = a + bu(o) \), your behavior will be unchanged

- Why would you want to maximize expected utility?
  - This is a question about preferences over lotteries
Compound lotteries

For two lottery tickets L and L’, let pL + (1-p)L’ be the “compound” lottery ticket where you get lottery ticket L with probability p, and L’ with probability 1-p.
Sufficient conditions for expected utility

- $L \geq L'$ means that $L$ is (weakly) preferred to $L'$
  - ($\geq$ should be complete, transitive)

- **Expected utility theorem.** Suppose
  - (continuity axiom) for all $L$, $L'$, $L''$, $\{p: pL + (1-p)L' \geq L''\}$ and $\{p: pL + (1-p)L' \leq L''\}$ are closed sets,
  - (independence axiom – more controversial) for all $L$, $L'$, $L''$, $p$, we have $L \geq L'$ if and only if $pL + (1-p)L'' \geq pL' + (1-p)L''$

then there exists a function $u: O \rightarrow \mathbb{R}$ so that $L \geq L'$ if and only if $L$ gives a higher expected value of $u$ than $L'$