Compiler Fall 2011

PRACTICE Midterm Exam

This is a full length practice midterm exam. If you want to take it at exam pace, give yourself 75 minutes to take the entire test. Just like the real exam, each question has a point value. There are 75 points in the exam, so that you can pace yourself to average 1 point per minute (some parts will be faster, some slower).

Questions:

1. SML [5 points]
2. Regular Languages [10 points]
3. NFA to regular expressions [10 points]
4. NFA to DFA [10 points]
5. Regular expression to NFA [10 points]
6. Context Free Langauges [10 points]
7. LL Parsing [10 points]
8. LR Parsing [10 points]

This is the solution set to the practice exam. The solutions appear in blue boxes.

Question 1: SML [5 pts]

Part A: Evaluate the following SML expressions (1 point each):

1. ```
   let val a = 4
       val b = 5
   in
     if (a > b) then 1 else 2
   end
   ```

   Answer:
   2

2. ```
   let val a = 2
       val b = let val a = 4
                   val q = 5
               in
                 a + q
               end
   in
     a + b
   end
   ```

   Answer:
   11

3. ```
   let fun f (0, b) = b
         | f (a, b) = f (a-1, a+b)
   in
       f (3, 5)
   end
   ```

   Answer:
   11
   ```

Part B: Write down the **type** for each of these SML functions (1 point each):

1. `fun f (x) = x ^ " " ^ x`

   Answer:
   string $\rightarrow$ string

2. ```
   fun f (x) = let fun g ([],  ans) = ans
                     | g (a::l,ans) = g(l, a::a::ans)
   in
     g(x,[])
   end
   ```

   Answer:
   'a list $\rightarrow$ 'a list
   ```

Question 2: Regular Languages [10 pts]

1. Write a regular expression for all strings of `a`s and `b`s which contains the substring `abba` (2 point).

   Answer:
   $(a \mid b)^*$ abba $(a \mid b)^*$

2. Write a regular expression for all strings of `x`s and `y`s where every `y` is immediately followed by at least 3 `x`s (2 points).

   Answer:
   $(x \mid (yxxx))^*$

3. Write a regular expression for all strings of `p`s and `q`s which contains an odd number of `q`s (3 points).

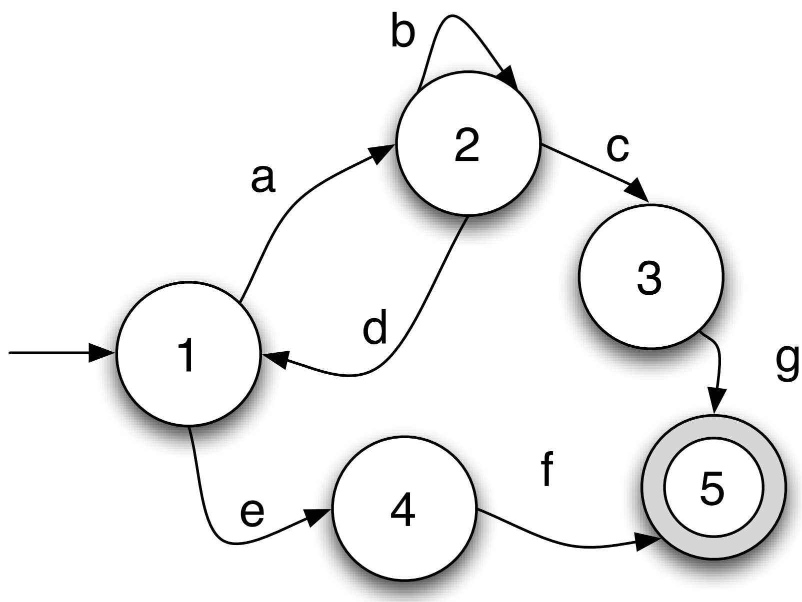
   Answer:
   $p^*\ q\ (\ (q\ p^*\ q) \mid p)^*$

4. A *finite langauge* is a language with a finite number of strings. For example, the language with only the string `a`, `ba`, and `bba` is finite, while the langauges in parts 1,2, and 3 above are not finite. Are all finite languages regular? If so, explain why. If not, give an example of a finite langauge which is not regular (3 points).

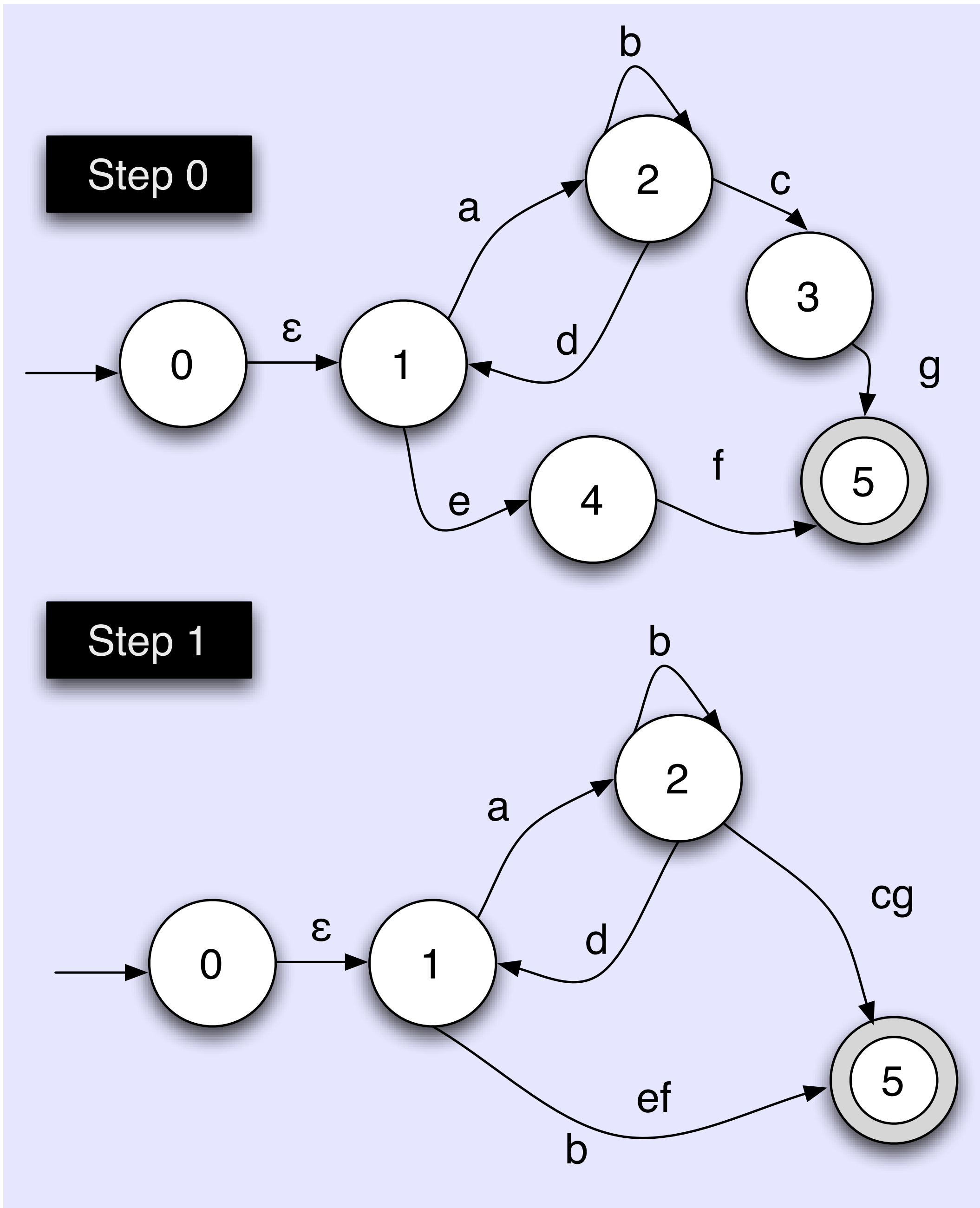
   Answer:
   Yes, all finite langauges are regular. Imagine a finite langauge comprised of strings s1, s2, ... sn. I can simply write a regular expression to describe it by ORing together all possible cases (s1) | (s2) | ... | (sn). Note that this only works for finite langauges, because I have a finite number of cases to OR together.

Question 3: NFA to Regexp [10 pts]

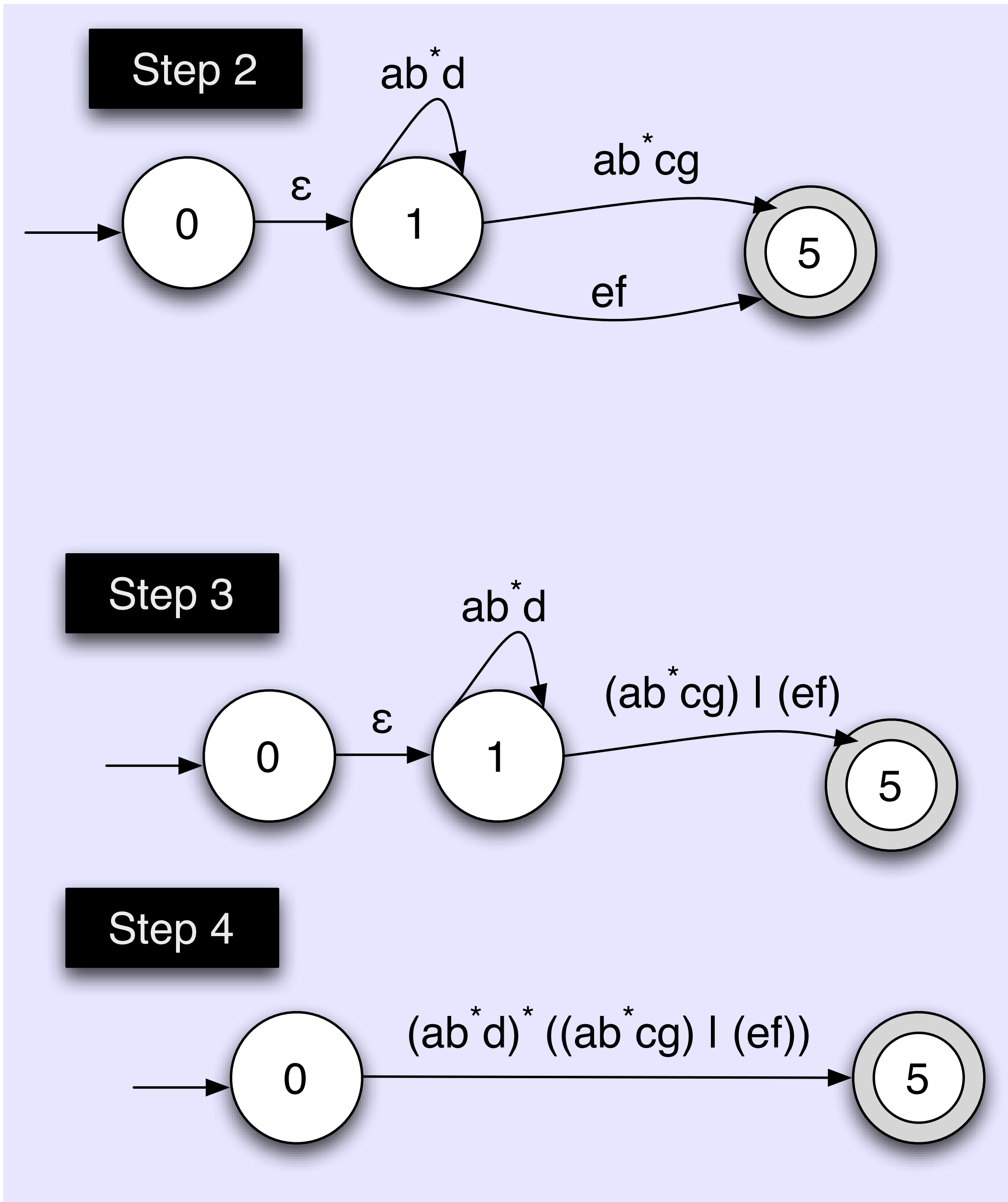
Convert the following NFA to a regular expression :



Answer:
The regular expression is $(ab^*d)^*$ ((ab^*cg) — (e f)). We derive it in 4 steps shown below (note you can remove states in different orders and come up with different, equivalent answers which are still correct).
Step 0: Put in a new start state with an ϵ transition to the old start. Step 1: Remove states 3 and 4. Step 2: Remove state 2. This results in a loop on 1 and a new edge from 1 to 5. Step 3: Collapse the parallel $1 \rightarrow 5$ edges into an OR. Step 4: Remove state 1. The answer is written on the remaining edge



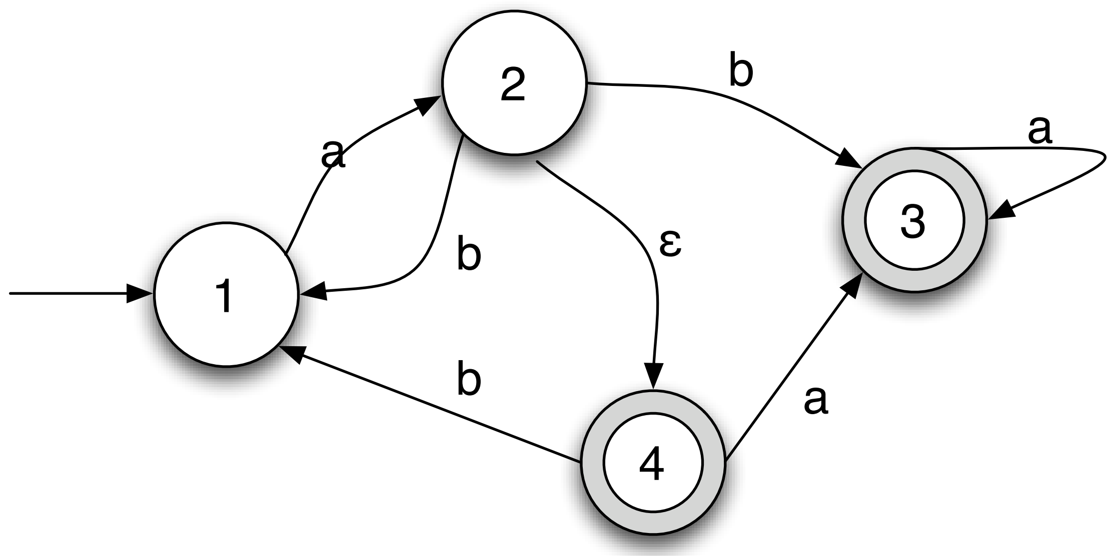
Workspace for question 3



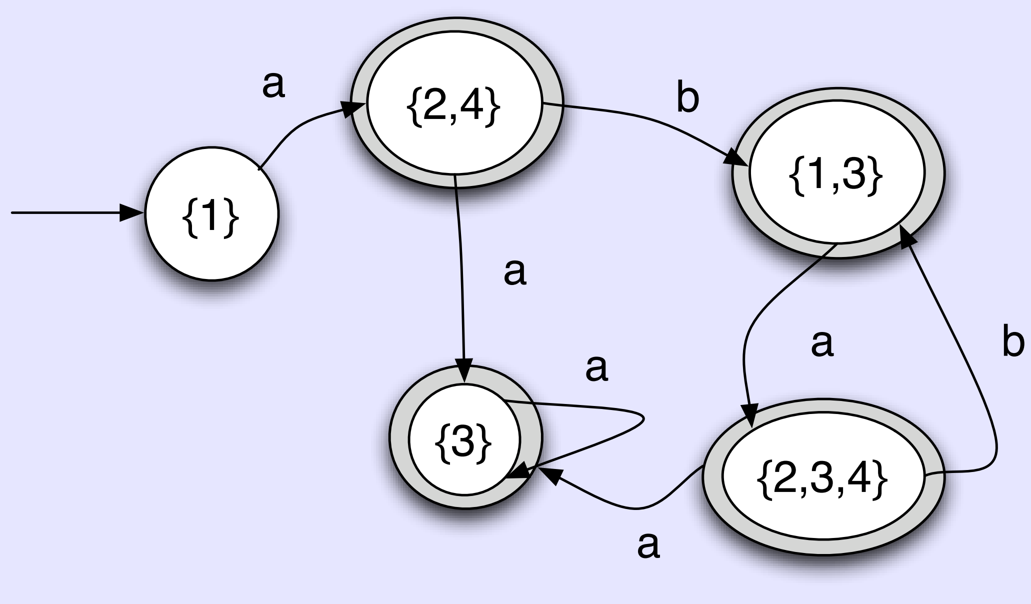
Workspace for question 3

Question 4: NFA to DFA [10 pts]

Convert the following NFA to a DFA:



Answer:

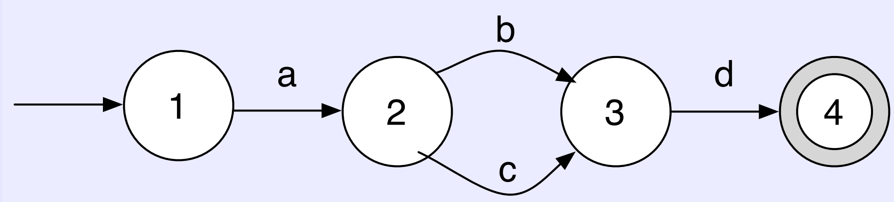


Question 5: Regexp to NFA [10 pts]

Draw an NFA for the following regular expressions:

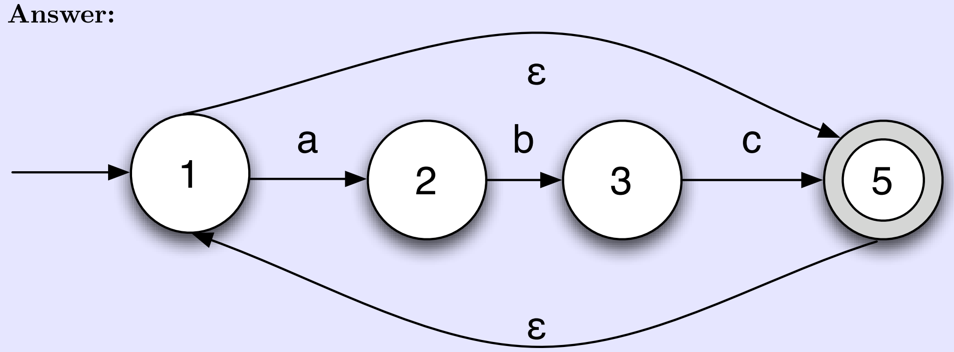
1. a (b |c) d (2 points)

Answer:



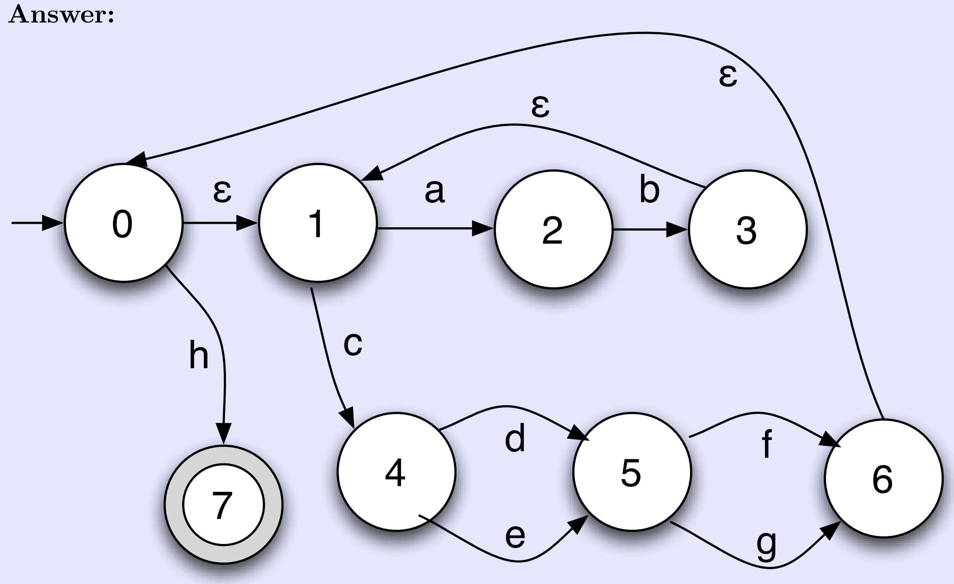
2. (abc)* (2 points)

Answer:



3. ((a b) * c (d | e) (f | g))* h (6 points)

Answer:



Question 6: Context Free Languages [10 pts]

Write context free grammars for the following languages (your grammar does not have to be LR(1), LL(1) etc):

1. All strings open and close parentheses, where the parentheses are balanced (2 points).

   Answer:
   S $\rightarrow$ (S)
   | S S
   $|\epsilon$

2. The language described by the regular expression $((ab)^*(c|d))^*$ (3 points).

   Answer:
   S $\rightarrow$ E S
   $|\epsilon$
   E $\rightarrow$ A c
   | A d
   A $\rightarrow$ a b A
   $|\epsilon$

   Note that this was derived by making A which is $(ab)^*$. Then making E which is the contents of the outer star. Then making S be zero or more repetitions of E.

3. Expressions consisting of `num`, `+`, and `*`. You should write your grammar so that `*` has higher precedence than `+` (5 points):

   Answer:
   E $\rightarrow$ E + T
   | T
   T $\rightarrow$ T * num
   | num

Question 7: LL Parsing [10 pts]

Consider the following grammar:

```
S -> S a S b
   | c
   | Q q
Q -> Q m
   |
```

- Which non-terminals (if any) can derive empty? (1 point)

  Answer:
  Q can derive empty.

- What are the FIRST sets of Q and S? (1 point)

  Answer:
  The FIRST set of Q is {m}. The FIRST set of S is {m,q,c}

  .

- What are the FOLLOW sets of Q and S? (1 point)

  Answer:
  The FOLLOW set of Q is {q,m}. The FOLLOW set of S is {a,b}.

- This grammar can not be parsed by an LL(0) or LL(1) parser. Explain why not (2 points).

  Answer:
  The grammar is left recursive (in both the rules S→S a S b and Q→ Q m). LL parsers cannot handle left recursion.

- Rewrite the grammar so that it accepts the same langauge, but can be parsed by an LL(1) parser (5 points).

  Answer:
  S → c Stail
  | Q q Stail
  Stail → a S b Stail
  |
  Q → m Q
  |

Question 8: LR Parsing [10 pts]

Consider the following grammar :

```
0: S -> X
1: X -> a X c
2: X -> X X
3: X -> b
```

1. What is Closure({X → X . X)? (2 points)

   Answer:
   X → X . X
   X → . a X c
   X → . X X
   X → . b

2. What is Goto({X → a . X c}, X)? (2 points)

   Answer:
   Closure{X → a X . c} which is just
   X → a X . c

3. Show the execution of the parser on the string `a b b c`. The state machine for the parser is provided along with a table for you to fill in on the next page (6 points).

Using the grammar from the previous page:

```
0: S -> X
1: X -> a X c
2: X -> X X
3: X -> b
```

And the state machine for that grammar:

State	a	b	c	$	S	X
1	s2	s3			g5	g4
2	s2	s3				g6
3	r3	r3	r3	r3		
4	s2	s3		r0		g7
5				Acpt		
6		s3	s8			g7
7	r2	r2	r2	r2		
8	r1	r1	r1	r1		

Fill in the table to the right. In one line, show the current status of the parser—the position in the input, the state the parser is in, and the contents of the stack. In the next line, show the action that the parser takes. Then show the new status in the following line. Repeat this process until the parser accepts the input. The first two are done for you.

Input	**State**	**Stack**
.a b b c	1	
Shift to state 2		
a. b b c	2	a_1
Shift to state 3		
a b . b c	3	a_1 b_2
Reduce by rule 3		
a b . b c	2	a_1 X_2
Goto 6		
a b . b c	6	a_1 X_2
Shift to state 3		
a b b . c	3	a_1 X_2 b_6
Reduce by rule 3		
a b b . c	6	a_1 X_2 X_6
Goto 7		
a b b . c	7	a_1 X_2 X_6
Reduce by rule 2		
a b b . c	2	a_1 X_2
Goto 6		
a b b . c	6	a_1 X_2
Shift to state 8		
a b b c .	8	a_1 X_2 c_6
Reduce by rule 1		
a b b c .	1	X_1
Goto 4		
a b b c .	4	X_1
Reduce by rule 0		
a b b c .	1	S_1
Goto 5		
a b b c .	5	S_1
Accept		