DNF Counting

Wednesday, October 02, 2013
2:28 PM

DNF Counting

DNF formula: Disjunction (OR) of clauses/conjunctions (ANDs)

Satisfiability of a DNF formula: Easy! Need to check if some clause can be satisfied

Q: How many truth assignments satisfy a given DNF formula?

Why can't we use the Monte Carlo method (counting via uniform sampling)?

Recall that # of samples $N \geq \frac{|U|}{|S|} \cdot \frac{1}{\varepsilon^2} \cdot \log(1/\delta)$

$|U| = 2^n$ for n variables

$|S|$ can be as small as ONE !!

So, we would exponentially many samples.

Solution: Importance sampling (Karp, Luby, Madras)

	C_1	C_2	C_3	$\cdots$	C_k
A_1	✓		✓		✓
A_2		✓			
$\vdots$					

Define U as the multi-set of all checks and S as the circled ones. Can we now use the Monte Carlo method?

- Drawing a uniform random sample from U:
  For each clause C_i, let N_i be the # of assignments satisfying it.
  Choose C_i w.p. $\frac{N_i}{\sum N_i}$. $\Longleftarrow$ A column chosen

Choose a satisfying assignment for C_i uniformly at random

- Checking if the sample is in S.

  Check the assignments in fixed order to determine if C_i is the first clause satisfied by the sample

- # of samples $= \frac{|U|}{|S|} \cdot \frac{1}{\varepsilon^2} \log(1/\delta) \leq k \cdot \frac{1}{\varepsilon^2} \cdot \log(1/\delta)$