Dual Fitting

- Analysis technique
- construct feasible dual with dual objective $\geq \frac{1}{\alpha}$ · algorithmic objective

$\Rightarrow$ Approx. factor of α.

Analysis of greedy algorithm for set cover:

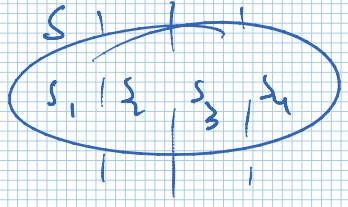
$$\min \sum_S c_S x_S \qquad \max \sum_{e \in U} y_e$$

$$\sum_{S : e \in S} x_S \geq 1 \qquad \sum_{e \in S} y_e \leq c_S$$

Dual: When you choose set S in the greedy algorithm which covers k_S new elements, set $y_e = \frac{c_S}{k_S \log n}$ on the new elements covered.

Lemma: The dual is feasible.

Proof:



$$\sum_{e \in S} y_e \leq \left(\frac{c_S}{|S|} + \frac{c_S}{|S - S_1|} + \cdots \right) \frac{1}{\log n}$$

$$\leq c_S \left(\frac{1}{n} + \frac{1}{n-1} + \cdots + 1 \right) \frac{1}{\log n}$$

$$\leq c_S \frac{\log n}{\log n} = c_S.$$