Due on December 5th, 2014
50 points total

General Directions: If you are asked to provide an algorithm, you should clearly define each step of the procedure, establish its correctness, and then analyze its overall running time. There is no need to write pseudo-code; an unambiguous description of your algorithm in plain text will suffice.

All the answers must be typed, preferably using LaTeX. If you are unfamiliar with LaTeX, you are strongly encouraged to learn it. However, answers typed in other text processing software and properly converted to a pdf file will also be accepted. Before submitting the pdf file, please make sure that it can be opened using any standard pdf reader (such as Acrobat Reader) and your entire answer is readable. **Handwritten answers or pdf files that cannot be opened will not be graded and will not receive any credit.**

Finally, please read the detailed collaboration policy given on the course website. You are **not** allowed to discuss homework problems in groups of more than 3 students. **Failure to adhere to these guidelines will be promptly reported to the relevant authority without exception.**

Problem 1 (25 points)
Your old friend Bob Bitfiddler is looking to open two new convience stores in the small town of Linesville, NC. Linesville is composed of a single east-west, ten-mile long street called Straight Street. There are n homes in total along Straight Street, and Bob has surveyed the residents of each of them in order to optimize the locations of his stores. For each house i located $\ell_i \in [0, 10]$ miles from the west gate of the town, his survey indicates there are r_i residents in home i, all of which are willing to walk at most $d_i \in [0, 10]$ miles to shop at one of Bob's two new stores. Bob's goal is to pick the locations of his two stores $s_1, s_2 \in [0, 10]$ along Straight Street that maximize the total number of residents that are willing to visit at least one of the stores.

Formulate the above problem as an integer programming problem (i.e., the optimal solution to your IP should solve for the two locations that maximize the total number of shoppers). Also provide a brief justification for why your formulation is correct.

Some notes:

- Although s_1 and s_2 could be fractional locations (i.e, they need not be placed at mile $1, 2, \ldots$), you can still restrict other variables to be integers since we are asking for an integer programming formulation.
- You do not need to write your IP in standard form, but the constraints and objective must be linear. For simplicity, you are permitted to use absolute values in your formulation (there are tricks for converting linear expressions with absolute values into standard form, but we would rather not have you complicate the formulation with these details).

Problem 2 (25 points)
Consider the following decision problems, SET COVER and VERTEX STEINER TREE:

SET COVER: You are given a set of n elements $U = \{x_1, \ldots, x_n\}$ and a collection of m sets $S = \{s_1, s_2, , \ldots, s_m\}$ such that each $s_i \subseteq U$ (i.e., each set in S is some subset of U). Given a fixed parameter K, determine whether it is possible to collectively include or *cover* all the elements in U with at most K sets from S. More formally, determine whether there exists a subset S' of S such that $\bigcup_{s \in S'} s = U$ and $|S'| \leq k$.

VERTEX STEINER TREE: You are given an unweighted, undirected graph $G = (V, E)$ and a set of terminal vertices $T \subseteq V$. Given a fixed parameter K, determine whether there exists a subgraph G' of G such G' connects all vertices in T (i.e., there exists a path between any $t_1, t_2 \in T$ in G') and includes at most K non-terminal vertices.

Given that SET COVER is NP-hard, prove that VERTEX STEINER TREE is NP-complete.