CPS 570: Artificial Intelligence

Decision theory

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Risk attitudes

• Which would you prefer?
  – A lottery ticket that pays out $10 with probability .5 and $0 otherwise, or
  – A lottery ticket that pays out $3 with probability 1

• How about:
  – A lottery ticket that pays out $100,000,000 with probability .5 and $0 otherwise, or
  – A lottery ticket that pays out $30,000,000 with probability 1

• Usually, people do not simply go by expected value

• An agent is risk-neutral if she only cares about the expected value of the lottery ticket

• An agent is risk-averse if she always prefers the expected value of the lottery ticket to the lottery ticket
  – Most people are like this

• An agent is risk-seeking if she always prefers the lottery ticket to the expected value of the lottery ticket
Decreasing marginal utility

- Typically, at some point, having an extra dollar does not make people much happier (decreasing marginal utility)
Maximizing expected utility

- Lottery 1: get $1500 with probability 1
  - gives expected utility 2
- Lottery 2: get $5000 with probability .4, $200 otherwise
  - gives expected utility .4*3 + .6*1 = 1.8
  - (expected amount of money = .4*$5000 + .6*$200 = $2120 > $1500)
- So: maximizing expected utility is consistent with risk aversion
Different possible risk attitudes under expected utility maximization

- **Green** has decreasing marginal utility $\rightarrow$ risk-averse
- **Blue** has constant marginal utility $\rightarrow$ risk-neutral
- **Red** has increasing marginal utility $\rightarrow$ risk-seeking
- **Grey’s** marginal utility is sometimes increasing, sometimes decreasing $\rightarrow$ neither risk-averse (everywhere) nor risk-seeking (everywhere)
What is utility, anyway?

• Function $u: O \rightarrow \mathbb{R}$ ($O$ is the set of “outcomes” that lotteries randomize over)

• What are its units?
  – It doesn’t really matter
  – If you replace your utility function by $u'(o) = a + bu(o)$, your behavior will be unchanged

• Why would you want to maximize expected utility?
  – This is a question about preferences over lotteries
Compound lotteries

- For two lottery tickets L and L’, let \( pL + (1-p)L' \) be the “compound” lottery ticket where you get lottery ticket L with probability \( p \), and L’ with probability \( 1-p \).

\[ p = 50\% \quad 1-p = 50\% \]

\[ pL+(1-p)L' \]

\[ \begin{array}{c}
  50\% \\
  25\% \\
  25\% \\
  75\% \\
  25\% \\
\end{array} \]

\[ \begin{array}{c}
  O_1 \\
  O_2 \\
  O_3 \\
  O_2 \\
  O_4 \\
\end{array} \]

\[ \begin{array}{c}
  25\% \\
  50\% \\
  12.5\% \\
  12.5\% \\
\end{array} \]

\[ \begin{array}{c}
  O_1 \\
  O_2 \\
  O_3 \\
  O_4 \\
\end{array} \]
Sufficient conditions for expected utility

• $L \geq L'$ means that $L$ is (weakly) preferred to $L'$
  – ($\geq$ should be complete, transitive)

• Expected utility theorem. Suppose
  – (continuity axiom) for all $L$, $L'$, $L''$, \{p: pL + (1-p)L' \geq L''\} and \{p: pL + (1-p)L' \leq L''\} are closed sets, and
  – (independence axiom – more controversial) for all $L$, $L'$, $L''$, $p > 0$, we have $L \geq L'$ if and only if $pL + (1-p)L'' \geq pL' + (1-p)L''$

Then, there exists a function $u: O \rightarrow \mathbb{R}$ so that $L \geq L'$ if and only if $L$ gives a higher expected value of $u$ than $L'$
Acting optimally over time

- **Finite** number of periods:
  - Overall utility = sum of rewards in individual periods

- **Infinite** number of periods:
  - … are we just going to add up the rewards over infinitely many periods?
    - Always get infinity!

- (Limit of) average payoff: \( \lim_{n \to \infty} \sum_{1 \leq t \leq n} r(t)/n \)
  - Limit may not exist…

- **Discounted** payoff: \( \sum_t \delta^t r(t) \) for some \( \delta < 1 \)

- Interpretations of discounting:
  - Interest rate \( r \): \( \delta = 1/(1+r) \)
  - World ends with some probability \( 1 - \delta \)

- Discounting is mathematically convenient