SQL: Recursion

Introduction to Databases
CompSci 316 Fall 2015
Announcements (Thu., Sep. 24)

• Homework #2 due in 1½ weeks
• Midterm in class in two weeks
  • Open-book, open-notes
  • Same format as sample midterm (from last year), to be posted on Sakai by Tuesday
A motivating example

**Example: find Bart’s ancestors**

**“Ancestor”** has a recursive definition

- *X* is *Y*’s ancestor if
  - *X* is *Y*’s parent, or
  - *X* is *Z*’s ancestor and *Z* is *Y*’s ancestor
Recursion in SQL

• SQL2 had no recursion
  • You can find Bart’s parents, grandparents, great grandparents, etc.
    
    ```sql
    SELECT p1.parent AS grandparent
    FROM Parent p1, Parent p2
    WHERE p1.child = p2.parent
    AND p2.child = 'Bart';
    ```
  • But you cannot find all his ancestors with a single query

• SQL3 introduces recursion
  • **WITH** clause
  • Implemented in PostgreSQL (**common table expressions**)
Ancestor query in SQL3

WITH RECURSIVE
Ancestor(anc, desc) AS
((SELECT parent, child FROM Parent)
UNION
(SELECT al.anc, a2.desc
FROM Ancestor al, Ancestor a2
WHERE al.desc = a2.anc))

SELECT anc
FROM Ancestor
WHERE desc = 'Bart';
Fixed point of a function

• If $f : T \rightarrow T$ is a function from a type $T$ to itself, a fixed point of $f$ is a value $x$ such that $f(x) = x$

• Example: What is the fixed point of $f(x) = x/2$?
  • 0, because $f(0) = 0/2 = 0$

• To compute a fixed point of $f$
  • Start with a “seed”: $x \leftarrow x_0$
  • Compute $f(x)$
    • If $f(x) = x$, stop; $x$ is fixed point of $f$
    • Otherwise, $x \leftarrow f(x)$; repeat

• Example: compute the fixed point of $f(x) = x/2$
  • With seed 1: 1, 1/2, 1/4, 1/8, 1/16, ... → 0

Doesn’t always work, but happens to work for us!
Fixed point of a query

• A query $q$ is just a function that maps an input table to an output table, so a **fixed point** of $q$ is a table $T$ such that $q(T) = T$

• To compute fixed point of $q$
  • Start with an empty table: $T \leftarrow \emptyset$
  • Evaluate $q$ over $T$
    • If the result is identical to $T$, stop; $T$ is a fixed point
    • Otherwise, let $T$ be the new result; repeat

☞ Starting from $\emptyset$ produces the unique minimal fixed point (assuming $q$ is monotone)
Finding ancestors

- WITH RECURSIVE Ancestor(anc, desc) AS
  ((SELECT parent, child FROM Parent)
   UNION
   (SELECT a1.anc, a2.desc
    FROM Ancestor a1,
    Ancestor a2
    WHERE a1.desc = a2.anc))
- Think of the definition as Ancestor = q(Ancestor)
Intuition behind fixed-point iteration

• Initially, we know nothing about ancestor-descendent relationships
• In the first step, we deduce that parents and children form ancestor-descendent relationships
• In each subsequent steps, we use the facts deduced in previous steps to get more ancestor-descendent relationships
• We stop when no new facts can be proven
Linear recursion

• With linear recursion, a recursive definition can make only one reference to itself

• Non-linear
  • `WITH RECURSIVE Ancestor(anc, desc) AS
    ((SELECT parent, child FROM Parent)
     UNION
     (SELECT al.anc, a2.desc
      FROM Ancestor al, Ancestor a2
      WHERE al.desc = a2.anc))`

• Linear
  • `WITH RECURSIVE Ancestor(anc, desc) AS
    ((SELECT parent, child FROM Parent)
     UNION
     (SELECT ________________
      FROM ________________
      WHERE ________________))`
Linear vs. non-linear recursion

- Linear recursion is easier to implement
  - For linear recursion, just keep joining newly generated Ancestor rows with Parent
  - For non-linear recursion, need to join newly generated Ancestor rows with all existing Ancestor rows

- Non-linear recursion may take fewer steps to converge, but perform more work
  - Example: \( a \rightarrow b \rightarrow c \rightarrow d \rightarrow e \)
  - Linear recursion takes 4 steps
  - Non-linear recursion takes 3 steps
    - More work: e.g., \( a \rightarrow d \) has two different derivations
Mutual recursion example

• Table *Natural* \((n)\) contains 1, 2, \ldots, 100

• Which numbers are even/odd?
  • An odd number plus 1 is an even number
  • An even number plus 1 is an odd number
  • 1 is an odd number

WITH RECURSIVE *Even*(n) AS
  (SELECT n FROM Natural
   WHERE n = ANY(SELECT n+1 FROM Odd)),
  RECURSIVE *Odd*(n) AS
  ((SELECT n FROM Natural WHERE n = 1)
   UNION
   (SELECT n FROM Natural
    WHERE n = ANY(SELECT n+1 FROM *Even*)))
Semantics of WITH

• WITH RECURSIVE $R_1$ AS $Q_1$, ..., RECURSIVE $R_n$ AS $Q_n$

$Q$;

• $Q$ and $Q_1, ..., Q_n$ may refer to $R_1, ..., R_n$

• Semantics

1. $R_1 \leftarrow \emptyset, ..., R_n \leftarrow \emptyset$

2. Evaluate $Q_1, ..., Q_n$ using the current contents of $R_1, ..., R_n$:
   $R_1^{new} \leftarrow Q_1, ..., R_n^{new} \leftarrow Q_n$

3. If $R_i^{new} \neq R_i$ for some $i$
   3.1. $R_1 \leftarrow R_1^{new}, ..., R_n \leftarrow R_n^{new}$
   3.2. Go to 2.

4. Compute $Q$ using the current contents of $R_1, ... R_n$ and output the result
Computing mutual recursion

WITH RECURSIVE Even(n) AS
  (SELECT n FROM Natural
   WHERE n = ANY(SELECT n+1 FROM Odd)),
RECURSIVE Odd(n) AS
  ((SELECT n FROM Natural WHERE n = 1)
   UNION
   (SELECT n FROM Natural
    WHERE n = ANY(SELECT n+1 FROM Even)))
• Even = ∅, Odd = ∅
• Even = ∅, Odd = {1}
• Even = {2}, Odd = {1}
• Even = {2}, Odd = {1, 3}
• Even = {2, 4}, Odd = {1, 3}
• Even = {2, 4}, Odd = {1, 3, 5}
• ...

Fixed points are not unique

WITH RECURSIVE
Ancestor(anc, desc) AS
((SELECT parent, child FROM Parent)
 UNION
 (SELECT al.anc, a2.desc
 FROM Ancestor al, Ancestor a2
 WHERE al.desc = a2.anc))

<table>
<thead>
<tr>
<th>parent</th>
<th>child</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homer</td>
<td>Bart</td>
</tr>
<tr>
<td>Homer</td>
<td>Lisa</td>
</tr>
<tr>
<td>Marge</td>
<td>Bart</td>
</tr>
<tr>
<td>Marge</td>
<td>Lisa</td>
</tr>
<tr>
<td>Abe</td>
<td>Homer</td>
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</tr>
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<td>Lisa</td>
</tr>
<tr>
<td>Ape</td>
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</tr>
</tbody>
</table>

Note how the bogus tuple reinforces itself!

• But if $q$ is monotone, then all these fixed points must contain the fixed point we computed from fixed-point iteration starting with $\emptyset$
  • Thus the unique minimal fixed point is the “natural” answer
Mixing negation with recursion

• If $q$ is non-monotone
  • The fixed-point iteration may flip-flop and never converge
  • There could be multiple minimal fixed points—we wouldn’t know which one to pick as answer!

• Example: popular users (pop $\geq 0.8$) join either Jessica’s Circle or Tommy’s
  • Those not in Jessica’s Circle should be in Tom’s
  • Those not in Tom’s Circle should be in Jessica’s
  • WITH RECURSIVE TommyCircle(uid) AS
    (SELECT uid FROM User WHERE pop $\geq 0.8$
    AND uid NOT IN (SELECT uid FROM JessicaCircle)),
  RECURSIVE JessicaCircle(uid) AS
    (SELECT uid FROM User WHERE pop $\geq 0.8$
    AND uid NOT IN (SELECT uid FROM TommyCircle))
Fixed-point iter may not converge

WITH RECURSIVE TommyCircle(uid) AS
(SELECT uid FROM User WHERE pop >= 0.8
 AND uid NOT IN (SELECT uid FROM JessicaCircle)),
RECURSIVE JessicaCircle(uid) AS
(SELECT uid FROM User WHERE pop >= 0.8
 AND uid NOT IN (SELECT uid FROM TommyCircle))

<table>
<thead>
<tr>
<th>uid</th>
<th>name</th>
<th>age</th>
<th>pop</th>
</tr>
</thead>
<tbody>
<tr>
<td>142</td>
<td>Bart</td>
<td>10</td>
<td>0.9</td>
</tr>
<tr>
<td>121</td>
<td>Allison</td>
<td>8</td>
<td>0.85</td>
</tr>
</tbody>
</table>
Multiple minimal fixed points

WITH RECURSIVE TommyCircle(uid) AS
  (SELECT uid FROM User WHERE pop >= 0.8
   AND uid NOT IN (SELECT uid FROM JessicaCircle)),
RECURSIVE JessicaCircle(uid) AS
  (SELECT uid FROM User WHERE pop >= 0.8
   AND uid NOT IN (SELECT uid FROM TommyCircle))

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Legal mix of negation and recursion

• Construct a dependency graph
  - One node for each table defined in WITH
  - A directed edge $R \rightarrow S$ if $R$ is defined in terms of $S$
  - Label the directed edge “−” if the query defining $R$ is not monotone with respect to $S$

• Legal SQL3 recursion: no cycle with a “−” edge
  - Called stratified negation

• Bad mix: a cycle with at least one edge labeled “−”

Legal!  
Illegal!

\[ \text{Ancestor} \quad \text{TommyCircle JessicaCircle} \]
Stratified negation example

• Find pairs of persons with no common ancestors

WITH RECURSIVE Ancestor(anc, desc) AS
((SELECT parent, child FROM Parent) UNION
 (SELECT a1.anc, a2.desc
  FROM Ancestor a1, Ancestor a2
  WHERE a1.desc = a2.anc)),

Person(person) AS
((SELECT parent FROM Parent) UNION
 (SELECT child FROM Parent)),

NoCommonAnc(person1, person2) AS
((SELECT p1.person, p2.person
  FROM Person p1, Person p2
  WHERE p1.person <> p2.person)
EXCEPT
 (SELECT a1.desc, a2.desc
  FROM Ancestor a1, Ancestor a2
  WHERE a1.anc = a2.anc))

SELECT * FROM NoCommonAnc;
Evaluating stratified negation

• The stratum of a node \( R \) is the maximum number of “—” edges on any path from \( R \) in the dependency graph
  • Ancestor: stratum 0
  • Person: stratum 0
  • NoCommonAnc: stratum 1

• Evaluation strategy
  • Compute tables lowest-stratum first
  • For each stratum, use fixed-point iteration on all nodes in that stratum
    • Stratum 0: Ancestor and Person
    • Stratum 1: NoCommonAnc

☞ Intuitively, there is no negation within each stratum
Summary

• SQL3 WITH recursive queries
• Solution to a recursive query (with no negation): unique minimal fixed point
• Computing unique minimal fixed point: fixed-point iteration starting from $\emptyset$
• Mixing negation and recursion is tricky
  • Illegal mix: fixed-point iteration may not converge; there may be multiple minimal fixed points
  • Legal mix: stratified negation (compute by fixed-point iteration stratum by stratum)