Indexing
Introduction to Databases
CompSci 316 Fall 2015
Announcements (Thu., Nov. 5)

• Project milestone #2 due today
• Homework #4 to be assigned next Tuesday
What are indexes for?

• Given a value, locate the record(s) with this value
  
  SELECT * FROM R WHERE A = value;
  SELECT * FROM R, S WHERE R.A = S.B;

• Find data by other search criteria, e.g.
  • Range search
    SELECT * FROM R WHERE A > value;
  • Keyword search
Dense and sparse indexes

- **Dense**: one index entry for each search key value
  - One entry may “point” to multiple records (e.g., two users named Jessica)
- **Sparse**: one index entry for each block
  - Records must be clustered according to the search key

<table>
<thead>
<tr>
<th>Sparse index on <em>uid</em></th>
<th>Dense index on <em>name</em></th>
</tr>
</thead>
<tbody>
<tr>
<td>123 Milhouse 10 0.2</td>
<td>Bart Jessica</td>
</tr>
<tr>
<td>142 Bart 10 0.9</td>
<td>Jessica</td>
</tr>
<tr>
<td>279 Jessica 10 0.9</td>
<td>Lisa</td>
</tr>
<tr>
<td>345 Martin 8 2.3</td>
<td>Martin</td>
</tr>
<tr>
<td>456 Ralph 8 0.3</td>
<td>Milhouse</td>
</tr>
<tr>
<td>512 Nelson 10 0.4</td>
<td>Nelson</td>
</tr>
<tr>
<td>679 Sherri 10 0.6</td>
<td>Ralph</td>
</tr>
<tr>
<td>697 Terri 10 0.6</td>
<td>Sherri</td>
</tr>
<tr>
<td>857 Lisa 8 0.7</td>
<td>Terri</td>
</tr>
<tr>
<td>912 Windel 8 0.5</td>
<td>Windel</td>
</tr>
<tr>
<td>997 Jessica 8 0.5</td>
<td>Jessica</td>
</tr>
</tbody>
</table>
Dense versus sparse indexes

• Index size
  • Sparse index is smaller

• Requirement on records
  • Records must be clustered for sparse index

• Lookup
  • Sparse index is smaller and may fit in memory
  • Dense index can directly tell if a record exists

• Update
  • Easier for sparse index
Primary and secondary indexes

• Primary index
  • Created for the primary key of a table
  • Records are usually clustered by the primary key
  • Can be sparse

• Secondary index
  • Usually dense

• SQL
  • PRIMARY KEY declaration automatically creates a primary index, UNIQUE key automatically creates a secondary index
  • Additional secondary index can be created on non-key attribute(s):
    CREATE INDEX UserPopIndex ON User(pop);
ISAM

• What if an index is still too big?
  • Put a another (sparse) index on top of that!

(ISAM (Index Sequential Access Method), more or less)

Example: look up 197
Updates with ISAM

Example: insert 107
Example: delete 129

- Overflow chains and empty data blocks degrade performance
  - Worst case: most records go into one long chain, so lookups require scanning all data!
**B⁺-tree**

- A hierarchy of nodes with intervals
- Balanced (more or less): good performance guarantee
- Disk-based: one node per block; large fan-out

Max fan-out: 4
Sample $B^+$-tree nodes

Max fan-out: 4

Non-leaf

120
150
180

to keys
$100 \leq k < 120$
$120 \leq k < 150$
$150 \leq k < 180$
$180 \leq k$

Leaf

120
130

to keys
$100 \leq k < 120$
$120 \leq k < 150$
$150 \leq k < 180$

Leaf

120
130

to next leaf node in sequence

to records with these $k$ values;
or, store records directly in leaves
**B⁺-tree balancing properties**

- Height constraint: all leaves at the same lowest level
- Fan-out constraint: all nodes at least half full (except root)

<table>
<thead>
<tr>
<th></th>
<th>Max # pointers</th>
<th>Max # keys</th>
<th>Min # active pointers</th>
<th>Min # keys</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-leaf</td>
<td>$f$</td>
<td>$f - 1$</td>
<td>$\lceil f/2 \rceil$</td>
<td>$\lceil f/2 \rceil - 1$</td>
</tr>
<tr>
<td>Root</td>
<td>$f$</td>
<td>$f - 1$</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Leaf</td>
<td>$f$</td>
<td>$f - 1$</td>
<td>$\lceil f/2 \rceil$</td>
<td>$\lceil f/2 \rceil$</td>
</tr>
</tbody>
</table>
Lookups

- \( \text{SELECT} \ * \ \text{FROM} \ R \ \text{WHERE} \ k = 179; \)
- \( \text{SELECT} \ * \ \text{FROM} \ R \ \text{WHERE} \ k = 32; \)
Range query

- \textbf{SELECT * FROM} \textit{R} \textbf{WHERE} \textit{k} > 32 \textbf{AND} \textit{k} < 179;

Max fan-out: 4

Look up 32...

And follow next-leaf pointers until you hit upper bound
Insertion

• Insert a record with search key value 32

Look up where the inserted key should go...

And insert it right there
Another insertion example

- Insert a record with search key value 152

Oops, node is already full!
Node splitting

Max fan-out: 4

Oops, that node becomes full!

Need to add to parent node a pointer to the newly created node
More node splitting

- In the worst case, node splitting can “propagate” all the way up to the root of the tree (not illustrated here)
  - Splitting the root introduces a new root of fan-out 2 and causes the tree to grow “up” by one level

Max fan-out: 4

Need to add to parent node a pointer to the newly created node
Deletion

- Delete a record with search key value 130

Max fan-out: 4

Look up the key to be deleted...

And delete it

Oops, node is too empty!

If a sibling has more than enough keys, steal one!
Stealing from a sibling

Max fan-out: 4

Remember to fix the key in the least common ancestor of the affected nodes
Another deletion example

• Delete a record with search key value 179

Max fan-out: 4

Cannot steal from siblings
Then coalesce (merge) with a sibling!
Coalescing

• Deletion can “propagate” all the way up to the root of the tree (not illustrated here)
  • When the root becomes empty, the tree “shrinks” by one level

Remember to delete the appropriate key from parent

Max fan-out: 4
Performance analysis

• How many I/O’s are required for each operation?
  • $h$, the height of the tree (more or less)
  • Plus one or two to manipulate actual records
  • Plus $O(h)$ for reorganization (rare if $f$ is large)
  • Minus one if we cache the root in memory

• How big is $h$?
  • Roughly $\log_{\text{fanout}} N$, where $N$ is the number of records
  • $B^+$-tree properties guarantee that fan-out is least $f/2$ for all non-root nodes
  • Fan-out is typically large (in hundreds)—many keys and pointers can fit into one block
  • A 4-level $B^+$-tree is enough for “typical” tables
B⁺-tree in practice

• Complex reorganization for deletion often is not implemented (e.g., Oracle)
  • Leave nodes less than half full and periodically reorganize
• Most commercial DBMS use B⁺-tree instead of hashing-based indexes because B⁺-tree handles range queries
The Halloween Problem

• Story from the early days of System R...

```
UPDATE Payroll
SET salary = salary * 1.1
WHERE salary >= 100000;
```

• There is a B⁺-tree index on Payroll(salary)
• The update never stopped (why?)

• Solutions?
B⁺-tree versus ISAM

• ISAM is more static; B⁺-tree is more dynamic
• ISAM can be more compact (at least initially)
  • Fewer levels and I/O’s than B⁺-tree
• Overtime, ISAM may not be balanced
  • Cannot provide guaranteed performance as B⁺-tree does
B+-tree versus B-tree

• B-tree: why not store records (or record pointers) in non-leaf nodes?
  • These records can be accessed with fewer I/O’s

• Problems?
Beyond ISAM, B-, and $B^+$-trees

- Other tree-based indexes: R-trees and variants, GiST, etc.
  - How about binary tree?

- Hashing-based indexes: extensible hashing, linear hashing, etc.

- Text indexes: inverted-list index, suffix arrays, etc.

- Other tricks: bitmap index, bit-sliced index, etc.