CompSci 527 Final Exam Sample— Sample Solution

IMPORTANT: Some of the solutions below contain explanations that were not asked for. In the exam, please be telegraphic but clear in your answers. Provide explanations only if they are asked for, or if you feel unsure about your answer, for partial credit.

1. This problem takes you through the computation of the set of all least-squares solutions to the following linear system:

$$\begin{aligned} 3x + 4y &= 2 \\ 3x + 4y &= 3 \end{aligned}$$

and the solutions to a related optimization problem. All the answers to the questions in this problem are numerical, and for the data given in the problem, and no more general answers are required. You may leave your answers in the form of fractions, with expressions like the following:

$$\frac{\sqrt{3}}{2} \begin{bmatrix} 2 \\ -5 \end{bmatrix} ,$$

but please simplify. If you cannot answer a question, write out a symbolic placeholder, and go on to the next question. For instance, if you don't know what $\mathbf{b}$ is in the first question, write

$$\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} .$$

(a) What are A and $\mathbf{b}$ if we write the system in this problem in the following form?

$$A\mathbf{x} = \mathbf{b}$$

Answer:

$$A = \begin{bmatrix} 3 & 4 \\ 3 & 4 \end{bmatrix} \quad \textit{and} \quad \mathbf{b} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} .$$

(b) What is the rank of A?
Answer: *1*

(c) Give a unit column vector $\mathbf{r}$ that spans the row space of A.
Answer:

$$\mathbf{r} = \frac{1}{5} \begin{bmatrix} 3 \\ 4 \end{bmatrix} .$$

(d) Give a unit column vector $\mathbf{n}$ that spans the null space of A.
Answer:

$$\mathbf{n} = \frac{1}{5} \begin{bmatrix} -4 \\ 3 \end{bmatrix} .$$

(e) Write the matrix V in the SVD $A = U\Sigma V^T$ of A.
Answer:

$$V = [\ \mathbf{r} \quad \mathbf{n}\ ] = \frac{1}{5} \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$$

(the sign of either column can be flipped for an equally acceptable answer).

(f) Compute the matrices U and Σ in the SVD of A. [Hint: compute $U\Sigma$ first.]
Answer:

$$U\Sigma = AV = \frac{1}{5} \begin{bmatrix} 3 & 4 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 25 & 0 \\ 25 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 5 & 0 \end{bmatrix}$$

so that the first column of U is $[1, 1]^T$ normalized to unit norm:

$$\mathbf{u}_1 = \frac{\sqrt{2}}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} .$$

The second column of U is orthogonal to the first and has unit norm, so

$$U = \frac{\sqrt{2}}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

(the sign of the second column can be flipped for an equally good answer, and the sign of the first column depends on that of $\mathbf{r}$*).*

The matrix Σ *is*

$$\Sigma = \sigma \begin{bmatrix} \sigma & 0 \\ 0 & 0 \end{bmatrix}$$

because A *is rank 1, and* σ *is* 5 *divided by the normalization factor of* $\mathbf{u}_1$*:*

$$\sigma = \frac{10}{\sqrt{2}} = 5\sqrt{2}\,,$$

so

$$\Sigma = \begin{bmatrix} 5\sqrt{2} & 0 \\ 0 & 0 \end{bmatrix}.$$

(g) Compute the pseudo-inverse $A^\dagger$ of A.

Answer:

$$A^\dagger = V\Sigma^\dagger U^T = \frac{1}{5}\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix} \frac{1}{5\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \frac{\sqrt{2}}{2} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 3 & 3 \\ 4 & 4 \end{bmatrix}.$$

(h) Find the minimum-norm solution $\mathbf{x}^*$ of the system $A\mathbf{x} = \mathbf{b}$.

Answer:

$$\mathbf{x}^* = A^\dagger \mathbf{b} = \frac{1}{50}\begin{bmatrix} 3 & 3 \\ 4 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 15 \\ 20 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 3 \\ 4 \end{bmatrix}.$$

(i) Give an expression for the set S of all least-squares solutions of the system $A\mathbf{x} = \mathbf{b}$.

Answer: *The set* S *is the minimum-norm solution plus any vector in the null space:*

$$S = \{\mathbf{x}^* \; + \; \alpha\mathbf{n}\} = \left\{ \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix} + \alpha \begin{bmatrix} -0.8 \\ 0.6 \end{bmatrix} \right\} = \left\{ \begin{bmatrix} 0.3 - 0.8\alpha \\ 0.4 + 0.6\alpha \end{bmatrix} \right\}$$

for any $\alpha \in \mathbb{R}$*.*

(j) Find all the solutions to

$$\hat{\mathbf{x}} = \arg\min_{\|\mathbf{x}\|=1} \|A\mathbf{x}\|\,.$$

Answer: *There are two solutions:*

$$\hat{\mathbf{x}} = \pm\mathbf{n} = \pm\begin{bmatrix} -0.8 \\ 0.6 \end{bmatrix}.$$

2. A one-dimensional, real image

$$I(x) \;:\; D \to \mathbb{R} \quad \text{where} \quad D \subset \mathbb{Z}$$

can be thought of as a single row out of a regular, two-dimensional, gray image. The integer domain D is the interval of pixel positions on which the image is defined. We want to track points between two such images

$$I(x) \;:\; D \to \mathbb{R} \quad \text{and} \quad J(x) \;:\; D \to \mathbb{R}$$

by finding the displacement d that minimizes the Sum-of-Squared-Differences (SSD) residual

$$\epsilon(x_I, d) = \sum_x [J(x+d) - I(x)]^2 \, w(x - x_I) \quad \text{where} \quad w(x) = \begin{cases} 1 & \text{if } |x| \le 3 \\ 0 & \text{otherwise} \end{cases}$$

is a window over 7 pixels, and x_I is the point in I we want to track. This problem is the one-dimensional formulation of point-feature tracking, and first and second derivative are analogous to gradient and Hessian for two-dimensional images.

(a) Write an expression for the Taylor series of $J(x+d)$ in d and around $J(x)$, truncated to just after the linear term. Use symbol $J'(x)$ to denote the derivative of $J(x)$ at x.

Answer:

$$J(x+d) \approx J(x) + J'(x)d\,.$$

(b) Use your answer to the previous question to write an expression that approximates $\epsilon(x_I, d)$ with a quadratic function of d.
Answer:

$$\epsilon(x_I, d) \approx \sum_x [J(x) + J'(x)d - I(x)]^2 \; w(x - x_I) \; .$$

(c) Write an expression for the approximate derivative $\epsilon'(x_I, d)$ with respect to d, using the quadratic function from the previous question as an approximation for ϵ.
Answer:

$$\epsilon'(x_I, d) \approx 2 \sum_x [J(x) + J'(x)d - I(x)] \; J'(x) \; w(x - x_I) \; .$$

(d) Write an expression for the approximate second derivative $\epsilon''(x_I, d)$ with respect to d, using the same approximation for ϵ as in the last question.
Answer:

$$\epsilon''(x_I, d) \approx 2 \sum_x [J'(x)]^2 \; w(x - x_I) \; .$$

(e) The Taylor series of $\epsilon(x_I, d)$ around 0 and truncated to the second term is

$$\epsilon(x_I, d) \approx \epsilon(x_I, 0) + \epsilon'(x_I, 0)d + \frac{1}{2}\epsilon''(x_I, 0)d^2 \; .$$

Find a formula for the minimum d^* of this truncated series with respect to d, assuming that $\epsilon''(x_I, 0) > 0$, and using the approximations for the derivatives of ϵ that you found when answering the last two questions.
Answer: *Since $\epsilon''(x_I, 0) > 0$, the unique minimum of this quadratic function of d can be found by setting its derivative to zero:*

$$\epsilon'(x_I, 0) + \epsilon''(x_I, 0)d = 0 \quad \Rightarrow \quad d^* = -\frac{\epsilon'(x_I, 0)}{\epsilon''(x_I, 0)}$$

so that

$$d^* = -\frac{\sum_x [J(x) - I(x)] \; J'(x) \; w(x - x_I)}{\sum_x [J'(x)]^2 \; w(x - x_I)} \; .$$

(f) Consider the trivial case in which I and J are shifted versions of the same linear function

$$I(x) = ax + b \quad \text{and} \quad J(x) = ax + c \; .$$

What do you expect the exact solution of the following optimization problem to be in this case, and for any x_I?

$$d_x = \arg\min_d \epsilon(x_I, d)$$

Answer: *We look for a displacement d_x such that*

$$J(x + d_x) = I(x) \; ,$$

that is,

$$a(x + d_x) + c = ax + b$$

so that

$$ad_x + c = b \quad \textit{and therefore} \quad d_x = \frac{b - c}{a} \; .$$

(g) Verify that in the special case described in question 2f the value you gave for d_x is the same as the value of d^* you found when answering question 2e.
Answer: *We have*

$$J(x) - I(x) = c - b \quad \textit{and} \quad J'(x) = a$$

so that

$$d^* = -\frac{\sum_x (c - b) \; a \; w(x - x_I)}{\sum_x a^2 \; w(x - x_I)} = -\frac{(c - b) \; a \; \sum_x \; w(x - x_I)}{a^2 \; \sum_x \; w(x - x_I)} = \frac{b - c}{a} = d_x \; .$$

(h) How many iterations would the one-dimensional version of the Lucas-Kanade tracker take to find the exact solution for the special case in question 2f, starting with initial solution $d_0 = 0$? Explain briefly and clearly why this is the case.

Answer: *The tracker would take one iteration, as my previous answer shows. The reason for this immediate convergence is that the images I and J are linear functions of x in the special case considered, and therefore the SSD residual $\epsilon(x_I, d)$ is an exact quadratic function of d. Lucas-Kanade approximates ϵ with a quadratic function at every iteration, but in this special case the approximation in the first iteration is exact. Therefore, the minimum of the approximation, which is found exactly, is also the exact minimum of ϵ.*

(i) The case $a = 0$ is problematic, because it would lead to division by zero. What is this problem called in computer vision?

Answer: *The aperture problem.*

(j) A good feature to track for the two-dimensional Lucas-Kanade tracker is one that satisfies the following constraints:

$$\kappa_2(A_I(\mathbf{x}_I)) \leq \kappa_{\max} \quad \text{and} \quad \sigma_{\min}(A_I(\mathbf{x}_I)) \geq \sigma_0$$

where $\kappa_{\max}$ and σ_0 are suitable thresholds and

$$A_I(\mathbf{x}_I) = \sum_{\mathbf{x}} \nabla I(\mathbf{x})[\nabla I(\mathbf{x})]^T \; w(\mathbf{x} - \mathbf{x}_I) \; .$$

Which of the two constraints above is relevant for the one-dimensional case examined in this problem, and what is the value of the left-hand side of the relevant constraint in the special case examined in question 2f?

Answer: *The condition number κ_2 for a 1×1 matrix is 1 if the only item in the matrix is nonzero, and is undefined otherwise, so the first constraint is useless. The second constraint is relevant, and*

$$A_I(x_I) = \sum_x [I'(x)]^2 \; w(\mathbf{x} - \mathbf{x}_I) \; .$$

The smallest singular value $\sigma_{\min}$ of $A_I(x_I)$ is the only singular value of the matrix, which is equal to the matrix itself. In the special case of question 2f we have

$$\sigma_{\min}(A_I(x_I)) = \sum_x a^2 \; w(\mathbf{x} - \mathbf{x}_I) = a^2 \sum_x w(\mathbf{x} - \mathbf{x}_I) = 7a^2$$

since the window w has seven values equal to 1 and the others are zero. This result is independent of x_I.

(k) What does the relevant constraint you identified in the previous question mean in terms of the images, for the special case in question 2f? Why is this constraint needed for good tracking?

Answer: *The constraint means that the functions I and J must have a slope that is sufficiently large in magnitude. If this were not the case, then the images would be close to constant, and so would be the SSD residual ϵ when viewed as a function of the displacement d. The displacement found by the tracker would then be mostly a function of image noise.*

3. This problem examines models for two distortion-free cameras, their essential matrix, and some aspects of the eight-point algorithm. Give all your answers in numerical form and for the specific cameras that are described in the first two questions of the problem, rather than in general terms.

(a) A certain camera C has no lens distortion, a focal length of 10 mm, a sensor with 200 pixels per millimeter both vertically and horizontally, and principal point at the pixel at column 600, row 400 in the image. Write the projection matrix P of the camera in homogeneous coordinates.

Answer:

$$P \sim K_s K_f \Pi = \begin{bmatrix} 200 & 0 & 600 \\ 0 & 200 & 400 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2000 & 0 & 600 & 0 \\ 0 & 2000 & 400 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} .$$

(b) A camera C' with the same projection matrix as camera C has its center of projection 100 millimeters to the right of camera C, along the X_1 axis of C. The optical axes of the two cameras are parallel to each other, their sensor rows are parallel to each other, and so are their sensor columns. Write the matrix G that transforms homogeneous coordinates $\mathbf{X}$ in the reference system of camera C to homogeneous coordinates $\mathbf{X}'$ in the reference system of camera C'.

Answer:

$$G \sim \left[\begin{array}{c|c} R & -R\mathbf{t} \\ \hline \mathbf{0}^T & 1 \end{array}\right] \quad \textit{where} \quad \mathbf{t} = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix} \quad \textit{and} \quad R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

so that

$$G \sim \begin{bmatrix} 1 & 0 & 0 & -100 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} .$$

(c) Write the projection matrix P' of camera C' so that if $\mathbf{X}$ is a vector of homogeneous coordinates of a point in the reference system of camera C then

$$\boldsymbol{\eta}' \sim P'\mathbf{X}$$

is a vector of homogeneous coordinates of the image point in camera C' (and in the reference system of C').

Answer:

$$P' \sim PG = \begin{bmatrix} 2000 & 0 & 600 & -200000 \\ 0 & 2000 & 400 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} .$$

(d) A point $\mathcal{X}$ in the world has *Euclidean* image coordinates

$$\tilde{\boldsymbol{\xi}} = \begin{bmatrix} 1200 \\ 800 \end{bmatrix} \quad \text{and} \quad \tilde{\boldsymbol{\eta}}' = \begin{bmatrix} 1000 \\ 800 \end{bmatrix}$$

(in pixels) in the images taken by the cameras C and C' described earlier. Find the *Euclidean* image coordinates $\tilde{\mathbf{x}} = e(\mathbf{x})$ and $\tilde{\mathbf{y}}' = e(\mathbf{y}')$ of $\mathcal{X}$ in the canonical reference system.

[Hints: Find homogeneous coordinates first. Also,

$$\begin{bmatrix} a & 0 & b \\ 0 & a & c \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{a} & 0 & -\frac{b}{a} \\ 0 & \frac{1}{a} & -\frac{c}{a} \\ 0 & 0 & 1 \end{bmatrix} .]$$

Answer: *From the hint, we see that the inverse of the internal calibration matrix for both cameras is*

$$K^{-1} = (K_s K_f)^{-1} = [P(:,1{:}3)]^{-1} \sim \begin{bmatrix} 2000 & 0 & 600 \\ 0 & 2000 & 400 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{2000} & 0 & -0.3 \\ 0 & \frac{1}{2000} & -0.2 \\ 0 & 0 & 1 \end{bmatrix}$$

so that the homogeneous coordinates of the canonical image points are

$$\mathbf{x} \sim K^{-1}\boldsymbol{\xi} \sim \begin{bmatrix} \frac{1}{2000} & 0 & -0.3 \\ 0 & \frac{1}{2000} & -0.2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1200 \\ 800 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.6 - 0.3 \\ 0.4 - 0.2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.3 \\ 0.2 \\ 1 \end{bmatrix}$$

and

$$\mathbf{y}' \sim K^{-1}\boldsymbol{\eta}' \sim \begin{bmatrix} \frac{1}{2000} & 0 & -0.3 \\ 0 & \frac{1}{2000} & -0.2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1000 \\ 800 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5 - 0.3 \\ 0.4 - 0.2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.2 \\ 1 \end{bmatrix} .$$

The Euclidean coordinates of the two canonical image points are therefore

$$\tilde{\mathbf{x}} = e(\mathbf{x}) = \begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix} \quad \textit{and} \quad \tilde{\mathbf{y}}' = e(\mathbf{y}') = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix} .$$

(e) The projection equations for camera C in its own canonical reference system are

$$\tilde{x}_1 = \frac{\tilde{X}_1}{\tilde{X}_3} \quad \text{and} \quad \tilde{x}_2 = \frac{\tilde{X}_2}{\tilde{X}_3}$$

in Euclidean coordinates. What are the projection equations for camera C' in the canonical reference system of camera C, for the particular geometry of the two cameras described in this problem? That is, what is the relationship between $\tilde{\mathbf{y}}'$ and $\tilde{\mathbf{X}}$? [Hint: There is no rotation between the cameras, so things are simple.]

Answer: *The transformation between the two cameras is a simple translation with no rotation, so*

$$\tilde{\mathbf{X}}' = \tilde{\mathbf{X}} - \mathbf{t} = \begin{bmatrix} \tilde{X}_1 - 100 \\ \tilde{X}_2 \\ \tilde{X}_3 \end{bmatrix}$$

and

$$\tilde{y}'_1 = \frac{\tilde{X}'_1}{\tilde{X}'_3} = \frac{\tilde{X}_1 - 100}{\tilde{X}_3} \quad \textit{and} \quad \tilde{y}'_2 = \frac{\tilde{X}'_2}{\tilde{X}'_3} = \frac{\tilde{X}_2}{\tilde{X}_3} \; .$$

(f) Solve the projection equations for the two cameras to find the vector $\tilde{\mathbf{X}}$ of the Euclidean coordinates of point $\mathcal{X}$ in the reference system of camera C. Specify the units of measurement for your solution.

Answer: *The projection equations can be rewritten as follows for the two given image points:*

$$\begin{aligned} 0.3\tilde{X}_3 &= \tilde{X}_1 \\ 0.2\tilde{X}_3 &= \tilde{X}_2 \\ 0.2\tilde{X}_3 &= \tilde{X}_1 - 100 \\ 0.2\tilde{X}_3 &= \tilde{X}_2 \; . \end{aligned}$$

The fourth equation is the same as the second, and is therefore ignored. Dividing the first equation by the second yields

$$\frac{0.3}{0.2} = \frac{\tilde{X}_1}{\tilde{X}_2} \quad \textit{that is,} \quad \tilde{X}_1 = 1.5\tilde{X}_2$$

and from the second and third equation we obtain

$$\tilde{X}_2 = \tilde{X}_1 - 100 \quad \textit{and therefore} \quad \tilde{X}_2 = 1.5\tilde{X}_2 - 100 \quad \textit{or} \quad 0.5\tilde{X}_2 = 100 \quad \textit{so that} \quad \tilde{X}_2 = 200 \textit{ mm}$$

and

$$\tilde{X}_1 = 1.5\tilde{X}_2 = 300 \textit{ mm} \quad , \quad \tilde{X}_3 = \frac{\tilde{X}_1}{0.3} = 1000 \textit{ mm.}$$

In summary, the Euclidean coordinates of point $\mathcal{X}$ are

$$\tilde{\mathbf{X}} = \begin{bmatrix} 300 \\ 200 \\ 1000 \end{bmatrix} \textit{mm.}$$

(g) Write an essential matrix E for this pair of cameras.

Answer: *Since there is no rotation, we have*

$$E \sim R[\mathbf{t}]_\times = [\mathbf{t}]_\times = \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -100 \\ 0 & 100 & 0 \end{bmatrix}$$

or more simply

$$E \sim \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

since nonzero multiplicative constants do not matter (any nonzero multiple of this matrix is an acceptable answer).

(h) Write the equation in canonical Euclidean coordinates of camera C of the epipolar line of the image point that has the following canonical Euclidean coordinates in the second image:

$$\tilde{\mathbf{y}}' = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix} \; .$$

Answer: *The homogeneous coordinates of the given point are*

$$\mathbf{y}' \sim \begin{bmatrix} 0.6 \\ 0.8 \\ 1 \end{bmatrix}$$

and the epipolar constraint then becomes

$$(\mathbf{y}')^T E\mathbf{x} = 0 \quad \textit{that is,} \quad \begin{bmatrix} 0.6 & 0.8 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \mathbf{x} = 0 \quad \textit{or} \quad \begin{bmatrix} 0 & 1 & -0.8 \end{bmatrix} \mathbf{x} = 0$$

so the epipolar line for that point has equation

$$x_2 - 0.8x_3 = 0$$

in homogeneous coordinates, or, setting $x_3 = 1$,

$$\tilde{x}_2 = 0.8$$

in Euclidean coordinates.

This makes sense: since the two optical axes are parallel to each other and perpendicular to the baseline, all epipolar lines are parallel to the image rows, and corresponding epipolar lines have the same $\tilde{x}_2$ *coordinate. So we could have found the required equation directly and without any computation, simply by reading the second coordinate of* $\tilde{\mathbf{y}}'$.

[This last explanation is not necessary for full credit. On the other hand, if you had used the explanation itself to give an answer without any calculations, you would have saved some time, and possibly earned some extra credit for your insight.]

(i) Let

$$\boldsymbol{\eta} = E(:)$$

be a vector of the entries of the essential matrix E you found above, listed in column-major order. For the two corresponding points with Euclidean coordinates $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{y}}'$ that you found when answering question 3d, write out the vector $\mathbf{a}$ such that the epipolar constraint for these two points has the form

$$\mathbf{a}^T \boldsymbol{\eta} = 0$$

suitable for use in the eight-point algorithm.

Answer: *The vector* $\mathbf{a}$ *is the Kronecker product of the homogeneous vectors* $\mathbf{x}$ *and* $\mathbf{y}'$:

$$\mathbf{a} = \mathbf{x} \otimes \mathbf{y}' = \begin{bmatrix} 0.3 \\ 0.2 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 0.2 \\ 0.2 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1\mathbf{y}' \\ x_2\mathbf{y}' \\ x_3\mathbf{y}' \end{bmatrix} = \begin{bmatrix} 0.06 & 0.06 & 0.3 & 0.04 & 0.04 & 0.2 & 0.2 & 0.2 & 1 \end{bmatrix}^T .$$

(Any nonzero multiple of this vector will do, since the equation $\mathbf{a}^T\boldsymbol{\eta} = 0$ *is homogeneous.)*