Decision Making for Robots and Autonomous Systems

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Utility Theory

Recall:

- We want to build rational agents
- Rational agents act to maximize performance

Performance is expressed using (equivalently):

- Object function
- Performance measure
- Performance metric
- Utility function

(presentation follows Ch. 3 in Kochenderfer)

Utility Theory

Most basic thing:

- Agent has to prefer some things over others.

Notion of *preference* over *events*:

- Prefer one *event* (or outcome) to another.
- Later: use these preferences to choose action.

Utility Theory

Some basic formal definitions of *preference*.

If we prefer B over A:

$$A \prec B$$

If we have no preference between A and B:

$$A \sim B$$

If we're not sure if we prefer B over A or have no preference:

$$A \preceq B$$

Utility theory

A *lottery* is defined by:

$$\{(E_1, p_1); (E_2, p_2); ...(E_n; p_n)\}$$

… a set of *events* (over which we have a preference), each with an associated *probability* of occurrence.

Naturally:

$$\sum_i p_i = 1$$

Utility theory

Neumann-Morgenstern axioms - posited for *rational* prefs.

Completeness: only one of $A \prec B, B \prec A, A \sim B$

Transitivity: if $A \preceq B$ and $B \preceq C$ then $A \preceq C$

Continuity: if $A \preceq B \preceq C$ then

$$\exists p \quad \{(A,p),(C,(1-p))\} \sim B$$

Independence: if $A \prec B$

$$\forall C, p \quad \{(A,p),(C,(1-p))\} \prec \{(B,p),(C,(1-p))\}$$

Utility theory

The von Neumann-Morgenstern axioms imply the existence of a **real-valued utility measure**, *U*.

There exists real-valued function U:

- $U : E \to \mathbb{R}$
- $U(A) < U(B)$ iff $A \prec B$
- $U(A) = U(B)$ iff $A \sim B$

U is unique up to an *affine transform:*

$$U_2 = mU_1 + c$$

for positive *m*.

Utility Theory

Now that we have a real-valued *U* …

$$U(\{(E_1, p_1); ...(E_n, p_n)\}) = \sum_i p_i U(E_i)$$

… *expected utility of a lottery*. This assigns an explicit utility value to an *uncertain event*.

The Maximum Expected Utility Principle

Every action results in a *lottery*.

The agent should select the outcome that leads to the lottery with maximum expected utility.

Let's say each action *a* leads to lottery outcome *o* with probability *P(o | a)*. The agent should select:

$$a^* = \max_a \sum_o P(o|a)U(o)$$

Variations of this are the whole core of rationality.

MEUP

What assumptions does this make?

Humans are Irrational

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Would you prefer:

- A: 100% change of losing 75 lives.
- B: 80% chance of losing 100 lives.

Most prefer B over A: $U(lose\ 75) < 0.8U(lose\ 100)$

Now:

- C: 10% chance of losing 75 lives.
- D: 8% chance of losing 100 lives.

Most prefer C over D: $0.1U(lose\ 75) > 0.08U(lose\ 100)$

[Tversky and Kahneman]

Humans are Irrational

Certainty effect:

- Humans exaggerate certain losses vs. probable losses.
- Exaggerate certain gains vs. probable gains.

Framing Effect

Two ways of phrasing the question:

Epidemic in town of 600 people.

E: 200 people will be saved.
F: 1/3 chance that 600 will be saved, 2/3 chance nobody saved.

People mostly choose E over F.

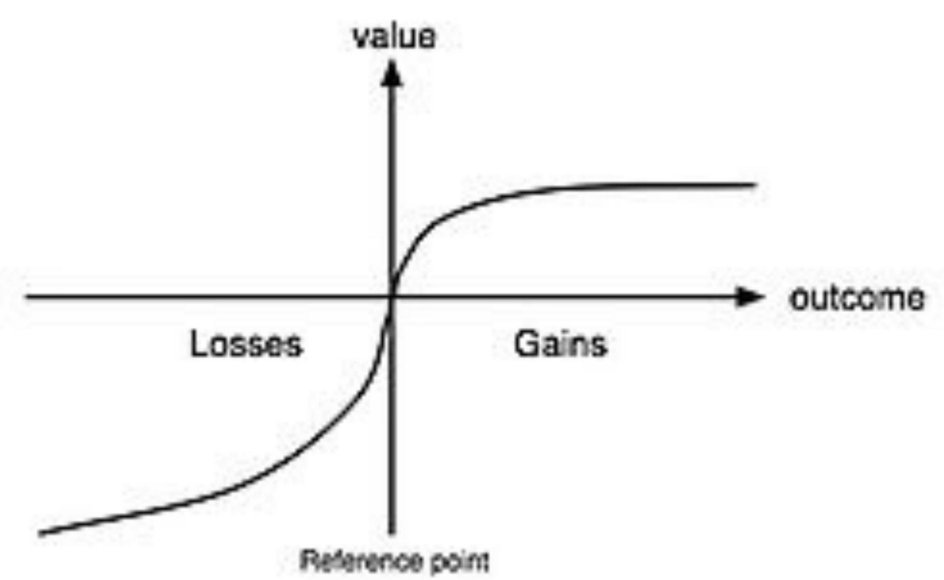
G: 400 people will die.
H: 1/3 chance that nobody dies, 2/3 chance that 600 die.

Majority if students choose H over G.

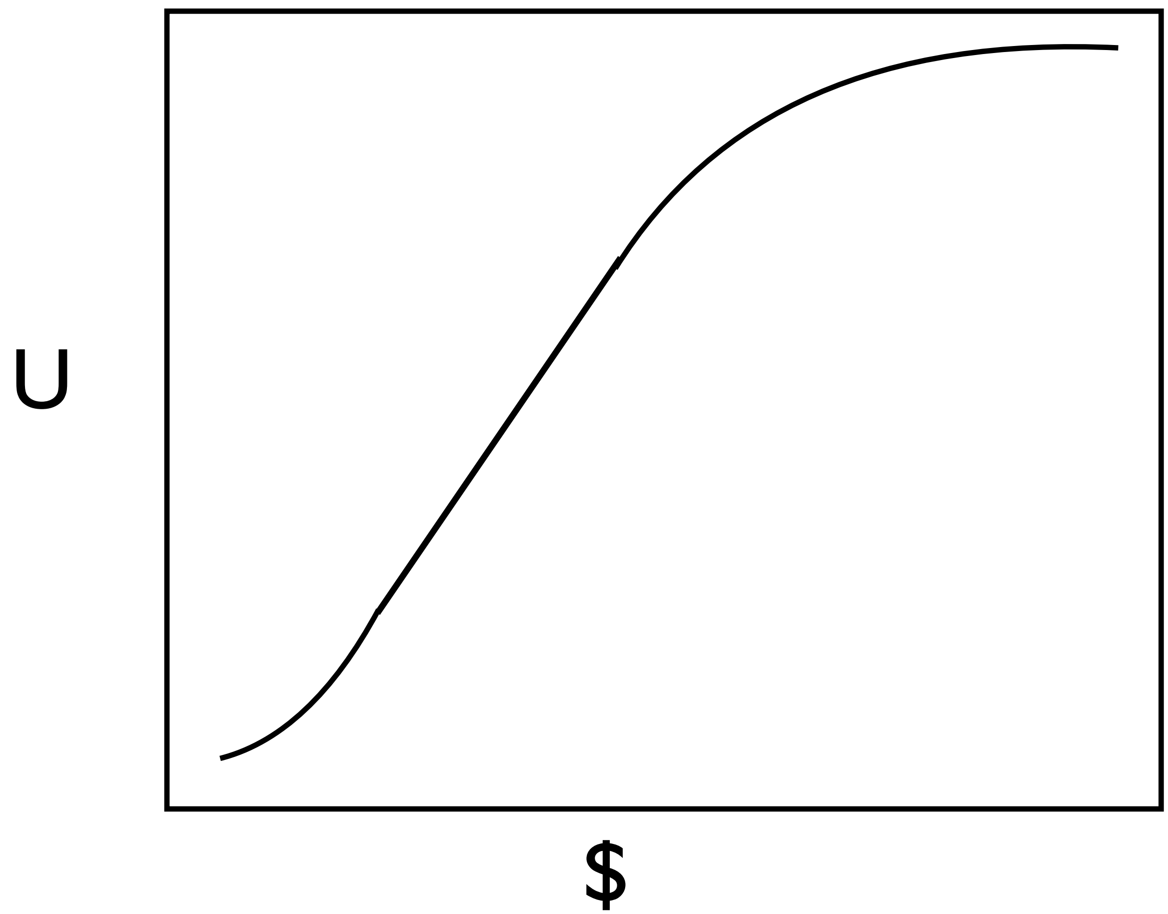
Prospect Theory

Leads to *prospect theory* for human decision-making:

- Set frame point.
- Utilities re-arranged around losses vs. gains.



The Value of Money



Risk neutral vs. averse vs. seeking

Assumptions

Thoughts on the assumptions here:

- The agent **knows** its utility function.
- The agent **knows U(E)**, for all events E.
- The agent has **just one** utility function.
- That utility function is **stationary**.

Thursday we look at the case where the agent does not know U(E) - and must learn it.