SQL: Recursion
Introduction to Databases
CompSci 316 Fall 2016

Announcements (Tue., Sep. 29)

- Homework #2 due tonight
- Deadline extended to Thursday for Problem 6 (Gradiance) and Problem X2 (non-Gradiance)
- Midterm in class Thursday
  - Open-book, open-notes
  - Same format as sample midterm (from last year)
    - Sample solution also posted in Sakai
- Project Milestone #1 due next Thursday

http://xkcdsw.com/1105
A motivating example

**Parent (parent, child)**

<table>
<thead>
<tr>
<th>Parent</th>
<th>Child</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homer</td>
<td>Bart</td>
</tr>
<tr>
<td>Homer</td>
<td>Lisa</td>
</tr>
<tr>
<td>Marge</td>
<td>Lisa</td>
</tr>
<tr>
<td>Abe</td>
<td>Homer</td>
</tr>
<tr>
<td>Ape</td>
<td>Abe</td>
</tr>
</tbody>
</table>

- Example: find Bart's ancestors
- "Ancestor" has a recursive definition
  - $X$ is $Y$'s ancestor if
    - $X$ is $Y$'s parent, or
    - $X$ is $Z$'s ancestor and $Z$ is $Y$'s ancestor

Recursion in SQL

- SQL2 had no recursion
  - You can find Bart's parents, grandparents, great-grandparents, etc.
    ```sql
    SELECT p1.parent AS grandparent
    FROM Parent p1, Parent p2
    WHERE p1.child = p2.parent
    AND p2.child = 'Bart';
    ```
  - But you cannot find all his ancestors with a single query
- SQL3 introduces recursion
  - WITH clause
  - Implemented in PostgreSQL (common table expressions)

**Ancestor query in SQL3**

```sql
WITH RECURSIVE Ancestor(anc, desc) AS
    ( (SELECT parent, child FROM Parent)
    UNION
    (SELECT a1.anc, a2.desc
    FROM Ancestor a1, Ancestor a2
    WHERE a1.desc = a2.anc)
    )
SELECT anc
FROM Ancestor
WHERE desc = 'Bart';
```

Query using the relation defined in WITH clause
Fixed point of a function

- If \( f: T \rightarrow T \) is a function from a type \( T \) to itself, a fixed point of \( f \) is a value \( x \) such that \( f(x) = x \).
- Example: What is the fixed point of \( f(x) = x/2 \)?
  - 0, because \( f(0) = 0/2 = 0 \).
- To compute a fixed point of \( f \):
  - Start with a "seed": \( x \leftarrow x_0 \).
  - Compute \( f(x) \).
    - If \( f(x) = x \), stop; \( x \) is fixed point of \( f \).
    - Otherwise, \( x \leftarrow f(x) \); repeat.
- Example: compute the fixed point of \( f(x) = x/2 \).
  - With seed 1: 1, 1/2, 1/4, 1/8, 1/16, ... \( \rightarrow 0 \).

Doesn’t always work, but happens to work for us!

Fixed point of a query

- A query \( q \) is just a function that maps an input table to an output table, so a fixed point of \( q \) is a table \( T \) such that \( q(T) = T \).
- To compute fixed point of \( q \):
  - Start with an empty table: \( T \leftarrow \emptyset \).
  - Evaluate \( q \) over \( T \).
    - If the result is identical to \( T \), stop; \( T \) is a fixed point.
    - Otherwise, let \( T \) be the new result; repeat.
- Starting from \( \emptyset \) produces the unique minimal fixed point (assuming \( q \) is monotone).

Finding ancestors

- WITH RECURSIVE
  - Ancestor (anc, desc) AS
    - (SELECT parent, child FROM Parent)
    - UNION
    - (SELECT a1.anc, a2.desc
      FROM Ancestor a1, Ancestor a2
      WHERE a1.desc = a2.anc)
- Think of the definition as \( \text{Ancestor} = q(\text{Ancestor}) \).

Homer Bart
Homer Lisa
Marge Bart
Marge Lisa
Abe Homer
Ape Abe
Abe Bart
Ape Abe
Ape Homer
Ape Lisa
Homer Bart
Homer Lisa
Marge Bart
Marge Lisa
Abe Homer
Abe Abe
Abe Bart
Abe Lisa
Ape Homer
Ape Lisa
Ape Abe
Ape Homer
Ape Lisa
Ape Abe
Ape Homer
Ape Lisa

Intuition behind fixed-point iteration

- Initially, we know nothing about ancestor-descendent relationships
- In the first step, we deduce that parents and children form ancestor-descendent relationships
- In each subsequent step, we use the facts deduced in previous steps to get more ancestor-descendent relationships
- We stop when no new facts can be proven

Linear recursion

- With linear recursion, a recursive definition can make only one reference to itself
- Non-linear
  - WITH RECURSIVE Ancestor(anc, desc) AS
    ((SELECT parent, child FROM Parent)
     UNION
     (SELECT a1.anc, a2.desc
      FROM Ancestor a1, Ancestor a2
      WHERE a1.desc = a2.anc))
- Linear
  - WITH RECURSIVE Ancestor(anc, desc) AS
    ((SELECT parent, child FROM Parent)
     UNION
     (SELECT anc, child
      FROM Ancestor, Parent
      WHERE desc = parent))

Linear vs. non-linear recursion

- Linear recursion is easier to implement
  - For linear recursion, just keep joining newly generated Ancestor rows with Parent
  - For non-linear recursion, need to join newly generated Ancestor rows with all existing Ancestor rows
- Non-linear recursion may take fewer steps to converge, but perform more work
  - Example: a → b → c → d → e
  - Linear recursion takes 4 steps
  - Non-linear recursion takes 3 steps
  - More work: e.g., a → d has two different derivations
Mutual recursion example

• Table `Natural (n)` contains 1, 2, ..., 100
• Which numbers are even/odd?
  • An odd number plus 1 is an even number
  • An even number plus 1 is an odd number
  • 1 is an odd number
WITH RECURSIVE `Even(n)` AS
  (SELECT n FROM `Natural` WHERE n = ANY(SELECT n+1 FROM `Odd`)),
RECURSIVE `Odd(n)` AS
  ((SELECT n FROM `Natural` WHERE n = 1)
   UNION
   (SELECT n FROM `Natural` WHERE n = ANY(SELECT n+1 FROM `Even`)))

Semantics of WITH

• WITH RECURSIVE `R_1` AS `Q_1`, ..., RECURSIVE `R_n` AS `Q_n` 
  `Q`;
  • `Q` and `Q_1` ..., `Q_n` may refer to `R_1`, ..., `R_n`
  • Semantics
    1. `R_1` ← ∅, ..., `R_n` ← ∅ 
    2. Evaluate `Q_1`, ..., `Q_n` using the current contents of `R_1`, ..., `R_n`:
      `R_i^{new}` ← `Q_i`, ..., `R_n^{new}` ← `Q_n`
    3. If `R_i^{new}` = `R_i` for some `i` 
      3.1. `R_i` ← `R_i^{new}`, ..., `R_n` ← `R_n^{new}` 
      3.2. Go to 2.
    4. Compute `Q` using the current contents of `R_1` ..., `R_n`
      and output the result
Computing mutual recursion

WITH RECURSIVE Even(n) AS
(SELECT n FROM Natural
WHERE n = ANY(SELECT n+1 FROM Odd)),
RECURSIVE Odd(n) AS
(SELECT n FROM Natural
WHERE n = 1)
UNION
(SELECT n FROM Natural
WHERE n = ANY(SELECT n+1 FROM Even))

• Even = Ø, Odd = Ø
• Even = Ø, Odd = {1}
• Even = {2}, Odd = {1}
• Even = {2}, Odd = {1, 3}
• Even = {2, 4}, Odd = {1, 3}
• Even = {2, 4}, Odd = {1, 3, 5}
• ...

Fixed points are not unique

WITH RECURSIVE Ancestor(anc, desc) AS
(SELECT parent, child FROM Parent)
UNION
(SELECT a1.anc, a2.desc
FROM Ancestor a1, Ancestor a2
WHERE a1.desc = a2.anc)

• But if q is monotone, then all these fixed points must contain the fixed point we computed from fixed-point iteration starting with Ø
• Thus the unique minimal fixed point is the "natural" answer

Mixing negation with recursion

• If q is non-monotone
  • The fixed-point iteration may flip-flop and never converge
  • There could be multiple minimal fixed points—we wouldn't know which one to pick as answer!

• Example: popular users (pop ≥ 0.8) join either Jessica’s Circle or Tommy’s
  • Those not in Jessica’s Circle should be in Tom’s
  • Those not in Tom’s Circle should be in Jessica’s
  • WITH RECURSIVE TommyCircle(uid) AS
      (SELECT uid FROM User WHERE pop >= 0.8
      AND uid NOT IN (SELECT uid FROM JessicaCircle)),
    RECURSIVE JessicaCircle(uid) AS
      (SELECT uid FROM User WHERE pop >= 0.8
      AND uid NOT IN (SELECT uid FROM TommyCircle))
Fixed-point iter may not converge

```
WITH RECURSIVE TommyCircle(uid) AS
  (SELECT uid FROM User WHERE pop >= 0.8
   AND uid NOT IN (SELECT uid FROM JessicaCircle)),
  RECURSIVE JessicaCircle(uid) AS
  (SELECT uid FROM User WHERE pop >= 0.8
   AND uid NOT IN (SELECT uid FROM TommyCircle))
```

Multiple minimal fixed points

```
WITH RECURSIVE TommyCircle(uid) AS
  (SELECT uid FROM User WHERE pop >= 0.8
   AND uid NOT IN (SELECT uid FROM JessicaCircle)),
  RECURSIVE JessicaCircle(uid) AS
  (SELECT uid FROM User WHERE pop >= 0.8
   AND uid NOT IN (SELECT uid FROM TommyCircle))
```

Legal mix of negation and recursion

- Construct a dependency graph
  - One node for each table defined in WITH
  - A directed edge $R \rightarrow S$ if $R$ is defined in terms of $S$
  - Label the directed edge “$-$” if the query defining $R$ is not monotone with respect to $S$
- Legal SQL3 recursion: no cycle with a “$-$” edge
  - Called stratified negation
- Bad mix: a cycle with at least one edge labeled “$-$”
Stratified negation example

- Find pairs of persons with no common ancestors

```
WITH RECURSIVE Ancestor(anc, desc) AS
  ((SELECT parent, child FROM Parent) UNION
   (SELECT a1.anc, a2.desc FROM Ancestor a1, Ancestor a2
    WHERE a1.desc = a2.anc)),

Person(person) AS
  ((SELECT parent FROM Parent) UNION
   (SELECT child FROM Parent)),

NoCommonAnc(person1, person2) AS
  ((SELECT p1.person, p2.person FROM Person p1, Person p2
    WHERE p1.person <> p2.person)
   EXCEPT
   (SELECT a1.desc, a2.desc FROM Ancestor a1, Ancestor a2
    WHERE a1.anc = a2.anc))

SELECT * FROM NoCommonAnc;
```

Evaluating stratified negation

- The stratum of a node R is the maximum number of 
  “−” edges on any path from R in the dependency graph
  - Ancestor: stratum 0
  - Person: stratum 0
  - NoCommonAnc: stratum 1

- Evaluation strategy
  - Compute tables lowest-stratum first
  - For each stratum, use fixed-point iteration on all nodes in that stratum
    - Stratum 0: Ancestor and Person
    - Stratum 1: NoCommonAnc

  Intuitively, there is no negation within each stratum

Summary

- SQL3 WITH recursive queries
- Solution to a recursive query (with no negation): unique minimal fixed point
- Computing unique minimal fixed point: fixed-point iteration starting from Ø
- Mixing negation and recursion is tricky
  - Illegal mix: fixed-point iteration may not converge; there may be multiple minimal fixed points
  - Legal mix: stratified negation (compute by fixed-point iteration stratum by stratum)