

1 Submodularity I

Is the set of submodular functions closed under addition? Prove or disprove.

2 Submodularity II

Is the set of submodular functions closed under multiplication? Prove or disprove.

3 Greedy Approximation

Consider a version of the set cover problem where the sets are partitioned into two types, red and blue. We still evaluate a cover by the number of atoms it covers, but we require that a cover use sets of only one color. If a cover uses sets of two different colors, it gets a score of zero. a) Explain precisely why the standard, greedy algorithm and analysis do not apply here. b) Provide a simple, polynomial time algorithm that *does* provide a reasonable performance guarantee.

4 Conditional Probability I

An airport terminal robot has been programmed to recognize dangerous devices and “disable” them. The robot is only used when there is a strong possibility that a dangerous object is present, and in such cases it is estimated that 2.1% of the objects it will analyze are indeed dangerous. In test cases, the robot has correctly identified dangerous objects as dangerous 87% of the time. However, it has also identified objects that are not dangerous as dangerous 5.2% of the time. What is the probability that an object identified as dangerous will actually be dangerous?

For the next three questions, we will use the following probability distribution:

Atomic Event	Probability
$P(\overline{abc})$	$\frac{1}{8}$
$P(\overline{a}bc)$	$\frac{1}{8}$
$P(\overline{a}\overline{b}\overline{c})$	$\frac{1}{6}$
$P(\overline{a}\overline{b}c)$	$\frac{1}{12}$
$P(\overline{a}b\overline{c})$	$\frac{3}{16}$
$P(\overline{a}bc)$	$\frac{3}{16}$
$P(a\overline{b}\overline{c})$	$\frac{1}{12}$
$P(abc)$	$\frac{1}{24}$

5 Conditional Probability II

Prove that C is conditionally independent of A given B . *Note: This is a little tedious because you need to compute a lot of things to prove this, but it's not hard. The fractions were chosen to make the arithmetic simple.*

6 Bayes Nets I

Provide the conditional probability tables for a Bayesian network for this distribution where variable C has B as its only parent, variable B has variable A as its only parent, and variable A has no parents.

7 Bayes nets II

Use variable elimination to compute $P(C)$ and verify that you get the same answer as what you get by summing the appropriate entries from the table above.