

1 Bayes Rule with Extra Evidence

Derive Bayes rule for variables A and B with extra evidence C :

$$P(A|BC) = \frac{P(B|AC)P(A|C)}{P(B|C)}$$

2 Monitoring

Reproduce the derivation of the monitoring equations from the class slides, filling in missing details. Each step should be justified using one of the following explanations:

- The Markov property
- Bayes rule with extra evidence: $P(A|BC) = \frac{P(B|AC)P(A|C)}{P(B|C)}$. In this case, indicate which variables are playing the roles of A , B , and C .
- The definition of conditional independence with extra evidence: $P(A|BC) = P(AB|C)/P(B|C)$. In this case, indicate which variables are playing the roles of A , B , and C .
- Marginalization

3 Smoothing

Reproduce the derivation of the smoothing equations from the class slides, filling in missing details. Each step should be justified using one of the following explanations:

- The Markov property
- Bayes rule with extra evidence: $P(A|BC) = \frac{P(B|AC)P(A|C)}{P(B|C)}$. In this case, indicate which variables are playing the roles of A , B , and C .
- The definition of conditional independence with extra evidence: $P(A|BC) = P(AB|C)/P(B|C)$. In this case, indicate which variables are playing the roles of A , B , and C .
- Marginalization

4 Prediction

You have been visiting your local farmers market almost every weekend for several years. You have noticed that one vendor who sells your favorite peaches is not there every week during peach season. You have noted that, if the vendor is present one week, there is a 70% chance that he will be present the following week. You have also noted that if the vendor is not present in a given week, then there is only a 50% chance that he will be present the week after.

This year, you purchased peaches from the vendor on the first market of peach season, but then you were out of town for two weeks on vacation. Now that you are back, you have a craving for peaches. What is the probability that the vendor will be present at the market this week (week 4)?

Show your work.

5 Tracking

(Continuing from the previous question) In order to determine if you should invite your friends over this weekend for your famous peach cobbler, you take to social media to try to better determine if the vendor will be at the market later this week. (Assume that the farmer's market is on Saturday morning.) You have noticed in the past that when he is present at the market, there is 95% probability that he will have posted something on social media in the preceding week. However, even when he isn't present at the market, there is still a 30% chance that he will have posted something. If he has posted on the weekdays leading up to weekends two, three, and four, what is the probability that the vendor will be present at the market this weekend (week 4)? Show your work.

6 Smoothing

(Continuing from the previous question) Compute the smoothed probability distribution over the vendor's status in week 2, taking into account all of the vendor's social media posts. Show your work.

7 Viterbi Path

(Continuing from the previous question) Compute the Viterbi path for weeks 2, 3, and 4. Your answer should be a single path through state space. Show your work.