COMPSCI 570 **Instructor:** Ronald Parr

Homework 6 **Due:** Tuesday, November 15, 2016

1 Regression

Suppose you have a regression problem with basis functions $\Phi = \phi_1 \ldots \phi_k$ and you have fit a data vector t to produce weight vector w using ordinary linear regression (L_2 error minimization, no regularization). Now suppose you create a new set of basis functions Φ' which is identical to Φ for all basis functions except ϕ'_i, where $\phi'_i = (\phi_i + c)$, where c is the same constant for all i. You can assume that $\phi_1 = 1$, i.e., ϕ_1 always returns value 1 for all inputs.

Prove that the fit using Φ' (to produce w') will be exactly the same on the training data, i.e., if w is the solution using Φ and w' is the solution using Φ', $\Phi w = \Phi' w'$.

Note that it is *not* sufficient simply to produce a w' for which $\Phi w = \Phi' w'$ if you do not also show that your w' is the correct solution for Φ'.

2 Linear Discriminants

Is the XOR function of two variables X_1 and X_2 linearly separable in the feature space X_1, X_2, $X_1 \text{AND} X_2$, and $X_1 \text{OR} X_2$)? Justify your answer.

3 Linear Discriminants II

In general, suppose you have a boolean function of two variables and four linearly independent features, will the boolean function be linearly separable in the feature space?

4 Multilayer Networks

Provide the architecture and weights for a multilayer neural network with a step activation function (SGN) that computes the XOR function.