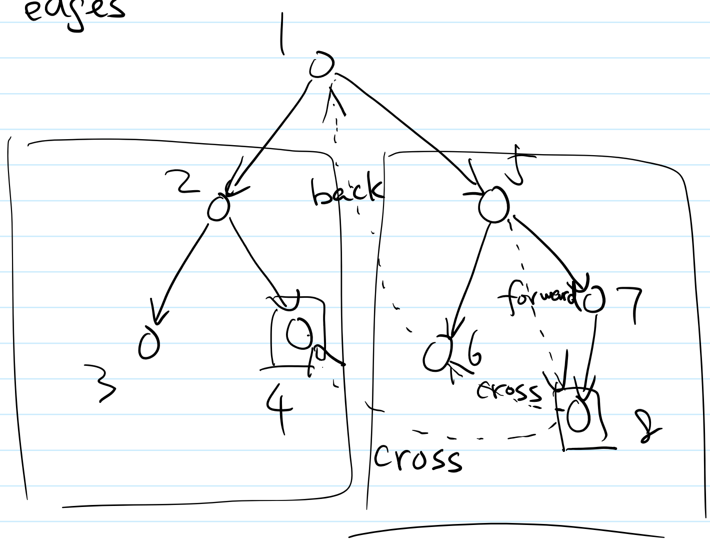
Lecture 12 Graph Algorithms II

Tuesday, October 17, 2017 2:48 PM

- Type of edges



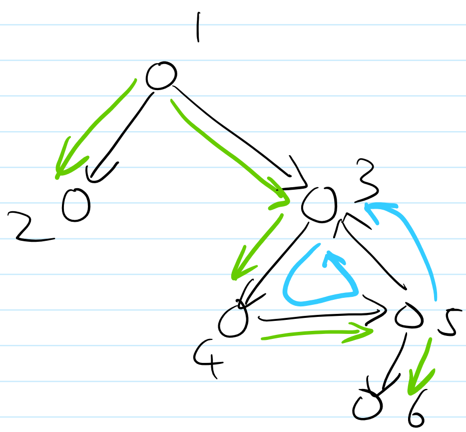
- running time of DFS

in DFS, visit every vertex and every edge once.

if graph is stored using adjacency list, DFS takes $O(n+m)$

#vertices #edges

- DFS cycle



- Claim: Graph G has a cycle, if and only if it has a back edge with respect to a DFS tree.

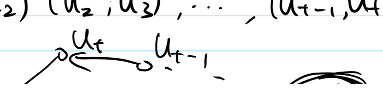
Proof: $\Leftarrow$ if there is a back edge (u, v)

by definition, there is a path from v to u using Tree edges

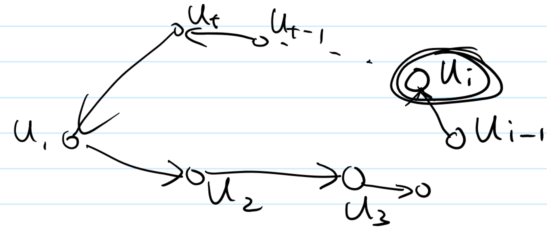
$(v, v_1, v_2, \ldots, u)$ this path + (u,v) forms a cycle

$\Rightarrow$ Let $(u_1, u_2, \ldots, u_t)$ be a cycle in graph G.

$((u_1,u_2) (u_2,u_3), \ldots, (u_{t-1},u_t), (u_t, u_1) \in E]$



$((u_1, u_2) (u_2, u_3), \cdots, (u_{t-1}, u_t), (u_t, u_1) \in E)$

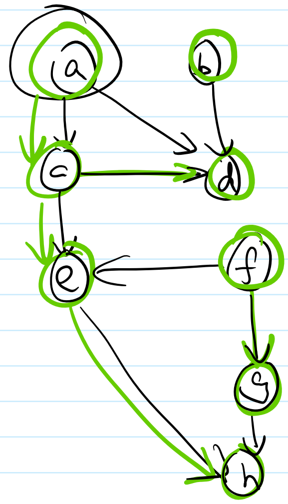


let u_i be the first vertex visited in this cycle

when u_{i-1} is visited, u_i must still be in stack

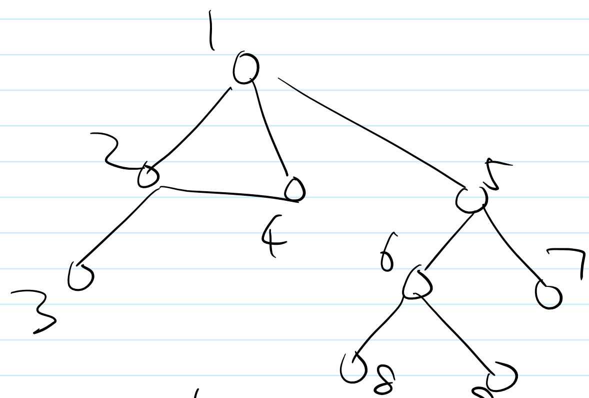
now (u_{i-1}, u_i) is a back edge $\square$

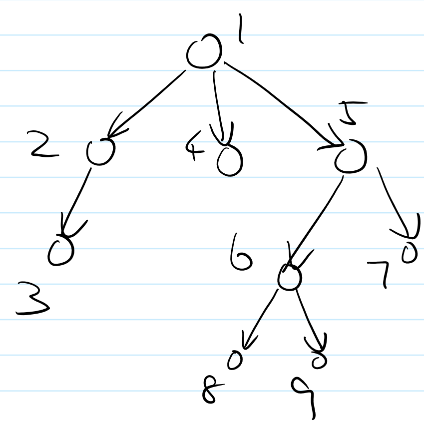
- Topological Sort



h e d c a b g f

- BFS





1 / 2 4 5 / 3 6 7 / 8 9

- shortest path property

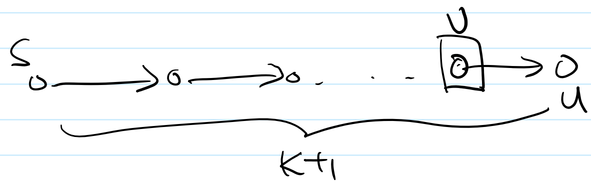
  - induction hypothesis: BFS visit all vertices of distance $\leq t$ before it visits any vertex of distance $t+1$.

base case: $t = 1$ easy (all neighbors are added to the queue at the beginning) induction step. assume IH is true for $t = k$

want to prove: BFS visits all vertices of distence $\le k+1$
before ——————— $k+2$.

Consider the time that the last vertex of distance k left the queue.

if u has distance $k+1$, then $\exists$ v s.t. distance of v is k
and (v, u) is an edge.



so v has been processed, and $\boxed{u \text{ is in the queue.}}$

if u has distance $\ge k+2$, all neighbors of u have distence $\ge k+1$
(predecessor)

u is not in the queue because of that.