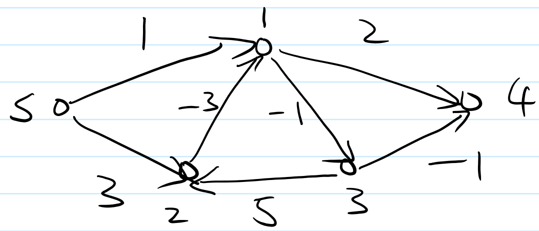
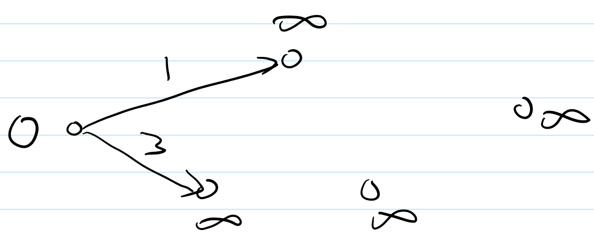
Lecture 14 Shortest Path (cont'd) Minimum Spanning Tree

Tuesday, October 24, 2017 2:21 PM

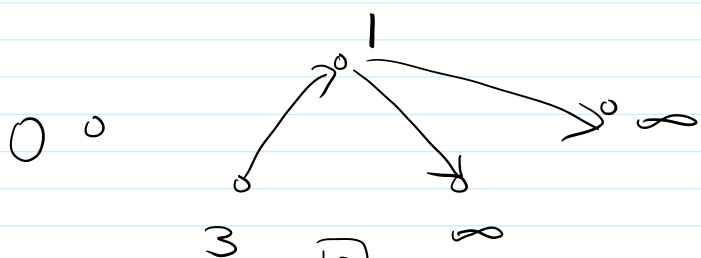
- shortest path with negative edge weights



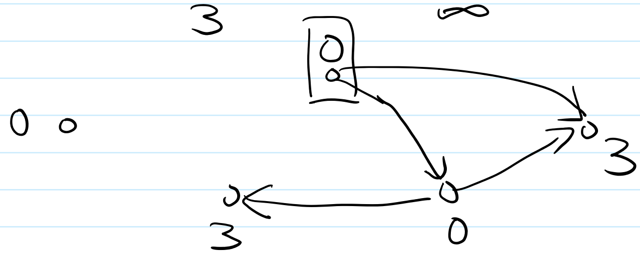
initial



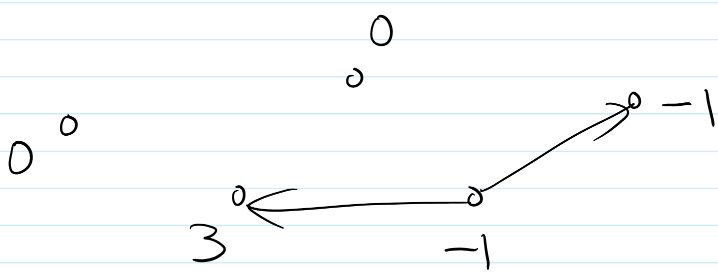
d[u, 0]



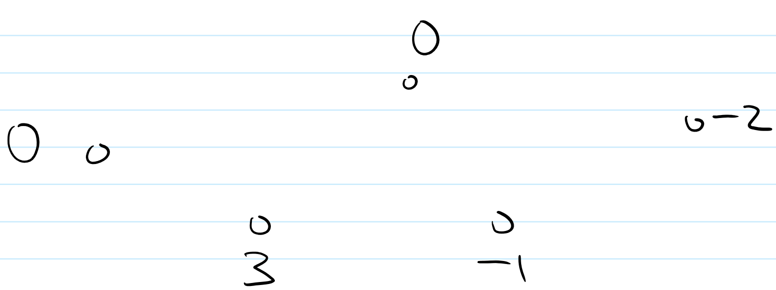
d[u, 1]



d[u, 2]



d[u, 3]



d[u, 4]

- Minimum Spanning Tree (MST)

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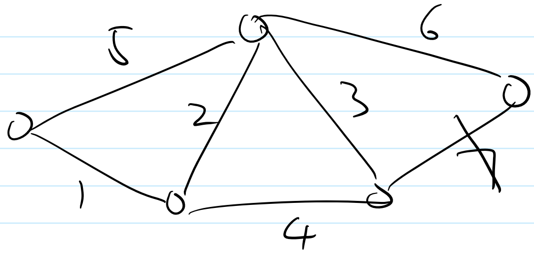
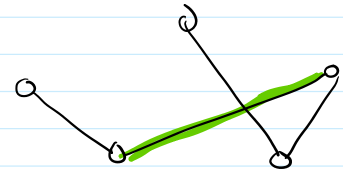
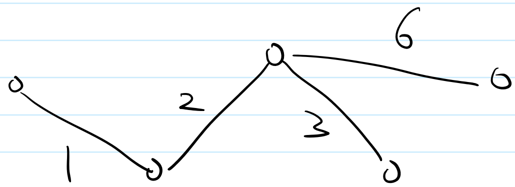
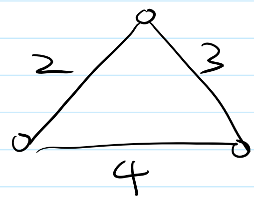




not spanning tree

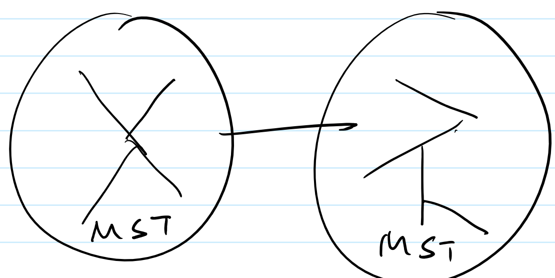


spanning tree

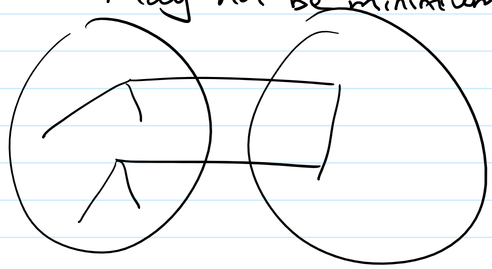


can always remove an edge from a cycle, maintaining connectivity of the graph

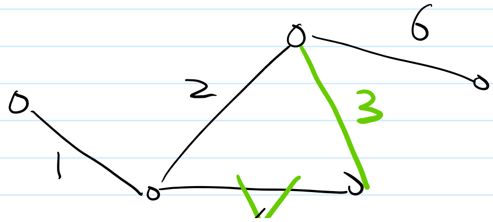
- property for MST
  - ① subtree of MST also a MST for the vertices it connects?
    - still true
    - hard to find an algorithm because we have 2^n subproblems



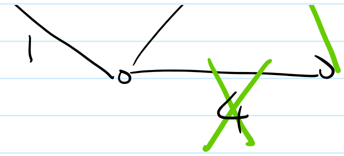
may not be minimum

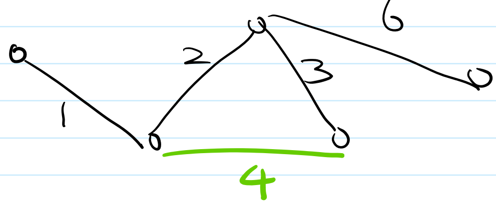


- swap operation



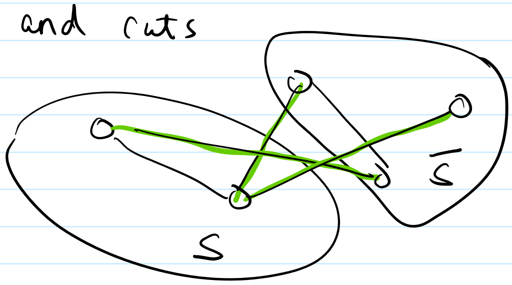
weight = 13

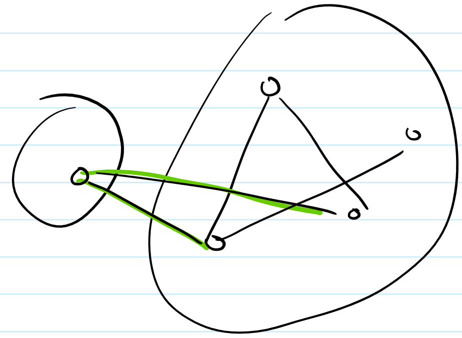




weight = 12

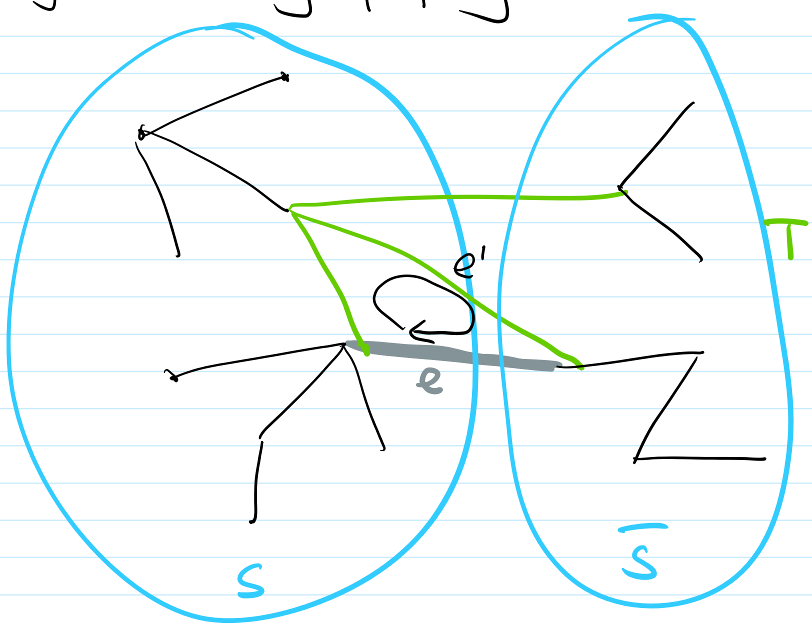
- graphs and cuts





Claim: in order to create a connected graph
we need to select at least 1 edge from every cut.
(same edge can serve multiple cuts)

- Proof of the key property



F back: edges we have decided on

green: additional edges to make F a MST

e: min cost edge between S and $\bar{S}$

if $e \in T$ (e is also green), trivial

if $e \notin T$ try a swap for $T \cup \{e\}$

$$w(e') \geq w(e)$$

$T' = T \cup \{e\} \setminus \{e'\}$ $w(T') \leq w(T)$ $\square$