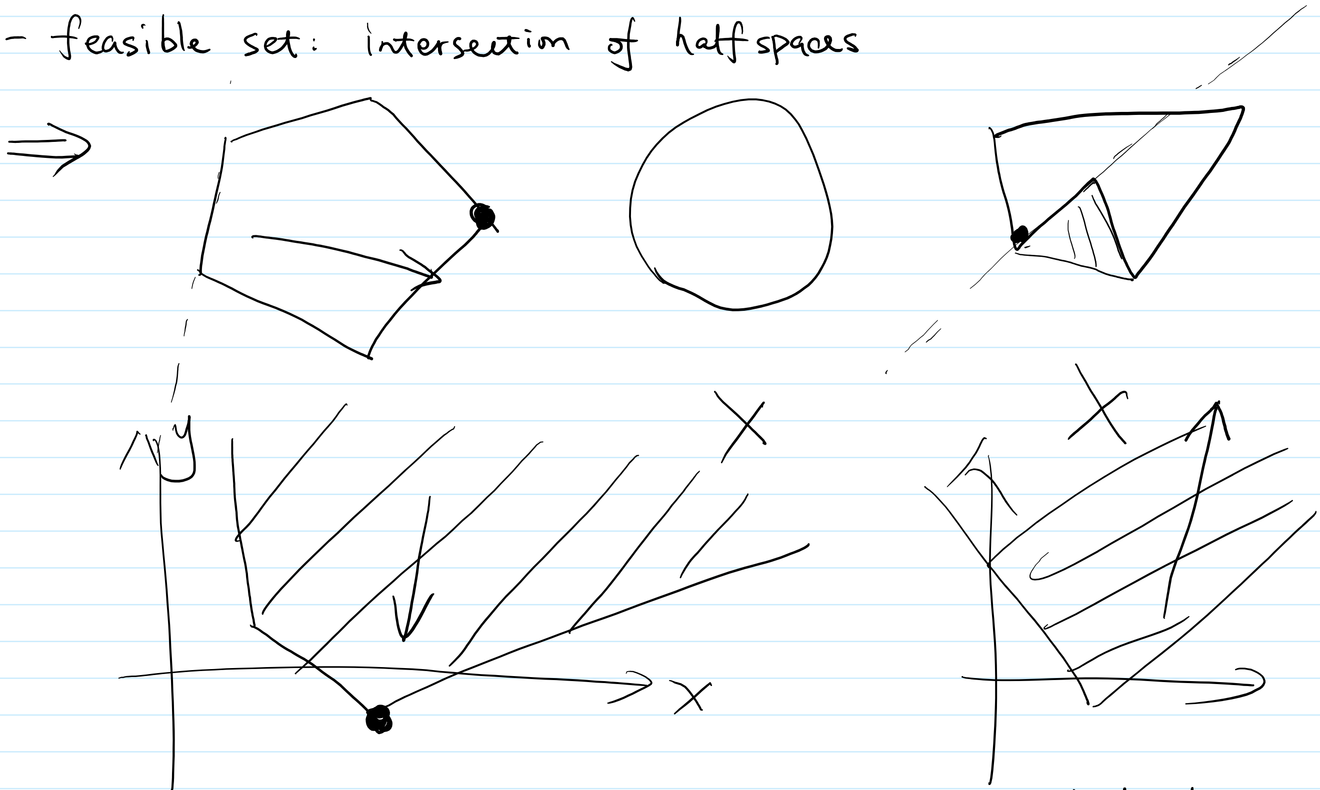
- Geometric Interpretation
  - feasible set: intersection of half spaces

$\Rightarrow$



no optimal solution
(optimal is ∞)

- Canonical Form

$$\min \langle c, x\rangle \quad \text{s.t.} \quad Ax \geq b, \quad x \geq 0$$

$c \quad (c_1, c_2, \ldots, c_n)$ constant

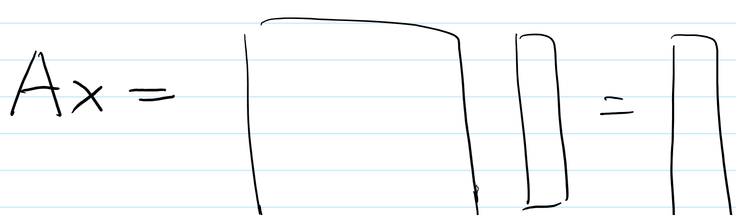
$x \quad (x_1, x_2, \ldots, x_n)$ variable

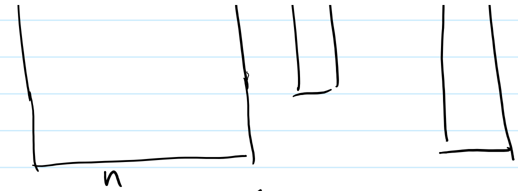
$$\langle c, x\rangle = \sum_{i=1}^{n} c_i x_i$$

$$A = \overbrace{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}}^{n} \Big\} m$$

$n =$ # variables

$m =$ # constraints





$$(Ax)_i = \sum_{j=1}^{n} a_{ij} x_j$$

$$b = (b_1, b_2, \dots, b_m)$$

$$x \geq 0 \Rightarrow x_1 \geq 0 \quad x_2 \geq 0 \quad \dots \quad x_n \geq 0$$

- convert LP to canonical form

$$\max \ 2x + y$$
$$x \geq 0$$
$$y \geq 0$$
$$x + y \leq 1$$

$\longrightarrow \vec{x} \geq 0$

$x_1 = x \quad x_2 = y$

$$\min \ -2x - y = \langle c, \vec{x} \rangle$$

$$c = (-2, -1)$$

$$x + y \leq 1 \Rightarrow \underbrace{-x - y}_{A\vec{x}} \geq \underbrace{-1}_{b}$$

$$A = (-1, -1) \qquad b = (-1)$$

$$\begin{aligned} &\min \ \langle c, \vec{x} \rangle \\ &\text{s.t.} \ A\vec{x} \geq b \\ &\quad \vec{x} \geq 0 \end{aligned} \iff \begin{aligned} &\min \ -2x_1 - x_2 \\ &-x_1 - x_2 \geq -1 \\ &x_1 \geq 0 \\ &x_2 \geq 0 \end{aligned}$$

$$\begin{aligned} &\min \ y + 2z \\ &\text{s.t.} \ z \geq 0 \\ &\quad y + z = 5 \end{aligned}$$

for y: $y = x_1 - x_2$

$x_1 \geq 0$

$x_2 \geq 0$

$z \qquad z = x_3$

$$\boxed{\begin{aligned} &\min \ (x_1 - x_2) + 2x_3 \\ &\text{s.t.} \ x_1, x_2, x_3 \geq 0 \\ &\quad (x_1 - x_2) + x_3 = 5 \end{aligned}}$$

$$x_1 - x_2 + x_3 \geq 5$$

$$x_1 - x_2 + x_3 \geq 5$$
$$x_1 - x_2 + x_3 \leq 5$$
$$-x_1 + x_2 - x_3 \geq -5$$

$$c = (1, -1, 2)$$

$$A = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \end{pmatrix} \quad b = \begin{pmatrix} 5 \\ -5 \end{pmatrix}$$

- Applying linear Program
  - matching

for each edge: have a variable $x_{i,j}$

$$x_{i,j} = \begin{cases} 1 & \text{if } (i,j) \text{ is matched} \\ 0 & \text{—— not matched.} \end{cases}$$

Constraints

① $0 \leq x_{i,j} \leq 1$ (ideally want $x_{i,j} = 0$ or 1)

② every classroom has 1 course

$$\forall j \quad \underbrace{\sum_{i:(i,j)\in E} x_{i,j}}_{\text{total \# Courses assigned to classroom } j.} \leq 1$$

③ every course has ≤ 1 classroom

$$\forall i \quad \underbrace{\sum_{j:(i,j)\in E} x_{i,j}}_{\text{total \# of classrooms that course } i \text{ is assigned to.}} \leq 1$$

Objective: want to match as many pairs as possible

$$\max \sum_{(i,j)\in E} x_{i,j}$$

- can use LP solver to find $x_{i,j}$
- if $x_{i,j} = 0/1$ for all $(i,j)\in E$, then it corresponds to a matching

if $x_{i,j}$ are fractional $\not\Rightarrow$ matching.

if $x_{ij} = 0/1$ for all $(i,j) \in E$, then it corresponds to a matching

if x_{ij} are fractional $\not\Rightarrow$ matching.

- for matching, can prove LP will always give integer solutions.