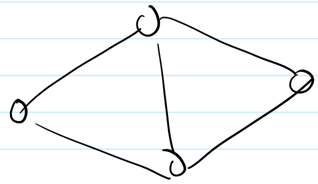
Lecture 23 NP-hardness reductions

Thursday, November 30, 2017 2:42 PM

- Reduction from Ind-Set to Clique.

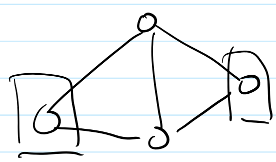


- similarity: both are graph problems

  both are looking for a set of vertices

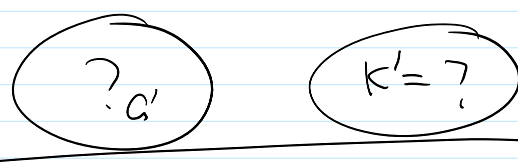
  edges in the set are opposite
  (no edges for ind-set
  all edges for clique)

- reduction: start with an input of original problem (Ind-Set)



$k = 2$

$\Longrightarrow$ convert to an input of the target problem (clique)

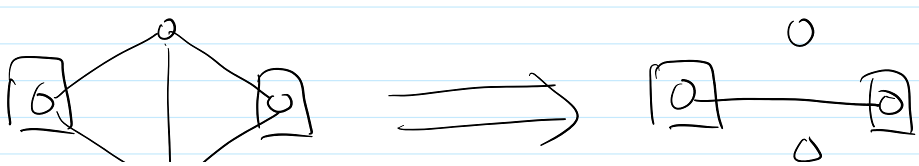


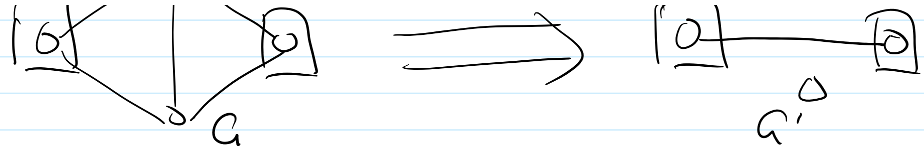
also: how to convert a solution to the original problem
to a solution to the target problem.

idea: maintain the solution set, but change the graph so
~~that~~ an Ind-Set becomes a clique.

- construct a new graph:

  if (u,v) was an edge, then $(u,v) \notin E$ in the new graph
  was not $\in$ ______





Claim: If S is an indep-set in G, then S is clique in G'.

(If Indep-Set has a solution/Yes $\Rightarrow$ Clique has a solution/Yes)

need the other direction: No $\Rightarrow$ No

for this direction we consider the contrapositive.

Claim: If S is a clique in G', then S is an Ind-Set in G.

Combining the claims: $\underbrace{\text{Ind-Set has a solution}}_{(G,k)} \Longleftrightarrow \underbrace{\text{Clique has a solution}}_{(G',k')}$

$(k'=k)$

- Summary: to do a reduction

① : convert input of original problem to input of the target

$$(G,k) \longrightarrow (G',k')$$

② if original input has answer Yes $\Longrightarrow$ the constructed input for target should have answer Yes

(usually: converting a solution for (G,k) of ind-set to a solution for (G,k') of Clique)

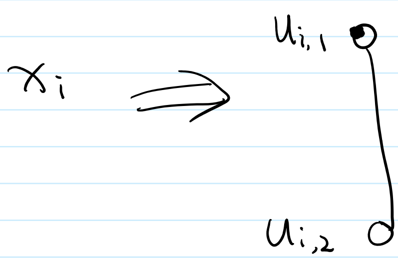
③ if constructed input Yes $\Longrightarrow$ original input Yes

(Convert a solution for (G',k') of clique to a solution for (G,k))

- 3-SAT to IND-SET

$$(x_1 \vee x_3 \vee \overline{x_5}) \wedge (x_2 \vee \overline{x}_3 \vee x_4) \wedge \cdots$$

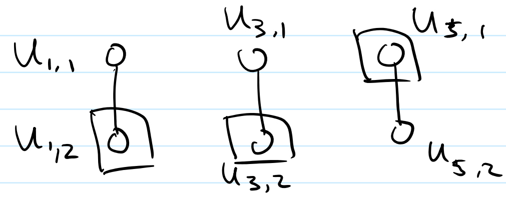
- variables



if $u_{i,1}$ is in ind-set, then $x_i = \text{true}$

$u_{i,2}$ is in ind-set, then $x_i = \text{false}$.

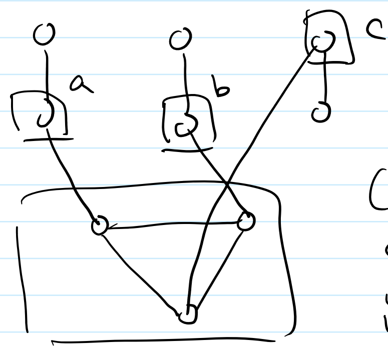
- clauses : $x_1 \vee x_3 \vee \overline{x_5}$



only bad case $x_1 =$ false $x_3 =$ false $x_5 =$ true

$\Longrightarrow$ $u_{1,2}$, $u_{3,2}$, $u_{5,1}$ are in ind-set.

- idea: construct a "gadget" that prevents all 3 of these vertices be in ind-set.



Claim: if a, b, c are all in ind-set cannot select any more vertex in square

if one of a, b, c is not in the set can select a vertex in the square.