- Example 1 LIS (Longest Increasing Subsequence)
  1. find subproblems and transition function

$\{4, 2, 3, 5, 1, 7, 10, \boxed{8}\}$

focus on last element of the array

is 8 in the LIS?

compare $\begin{cases} \text{① Length of LIS if 8 is not in the sequence} \\ \text{② \_\_\_\_\_\_\_\_ if 8 is in the sequence.} \end{cases}$

- to solve ① call LIS on $\{4, 2, 3, \ldots, 10\}$
- to solve ②, not as easy
  - in this example LIS $\{4, 2, 3, \ldots 10\} = \{2, 3, 5, 7, 10\}$
    cannot add 8 to this sequence
  - $\{2, 10, 3, 5, 8\}$
    LIS $\{2, 10, 3, 5\} = \{2, 3, 5\}$
    can/should add 8 to this sequence.
- ideas: (a): have a subproblem $a[i, j]$
  s.t. $a[i, j]$ = the LIS of first i elements whose last element is smaller than j

(b) have a subproblem $a[i]$
  $a[i]$ = the LIS of first i elements that ends at i-th element

A	4	2	3	5	1	7	10	8
a	1	1	2	3	1	4	5	5

how to compute $a[i]$? i-th element must be in

if $j < i$ $A[j] < A[i]$ then we can add $A[i]$ to the end of an IS ending at $A[j]$

$$a[i] = \begin{cases} 1 & \text{if } A[i] < A[j] \text{ for all } j<i \\ \max\limits_{\substack{j<i \\ A[j]<A[i]}} a[j]+1 \end{cases}$$

② figure out the base cases

③ find an appropriate order $i = 1, 2, 3, \ldots, n$

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for i = 1 to n
    a[i] = 1
    for j = 1 to i-1
        if A[j] < A[i] and a[j]+1 > a[i] Then
            a[i] = a[j]+1
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Example 2 Knapsack

Look at the last item

① max value if last item is in Knapsack
② ______ if last item is not in Knapsack.

- to solve ① w_n, V_n already in my Knapsack
  should try to maximize value for first $n-1$ items using a Knapsack of capacity $W - w_n$.
  ② should try to maximize value for first $n-1$ items using a Knapsack of capacity W.

$a[i,j]$ = max value from first i items with capacity j.

① $a[n-1, W-w_n]$
② $a[n-1, W]$

$$\boxed{a[i,j] = \max \begin{cases} a[i-1, j-W_i] + V_i & \text{putting ith element in} \quad j \geq W_i \\ a[i-1, j] & \text{not putting ith element} \end{cases}}$$

$a[0, \text{anything}] = 0$ (no items)

$a[i, 0] = 0$ (no capacity)

ordering $i = 1$ to n

$j = 1$ to W

$a[i,j] = a[i-1,j]$

if $j \geq w_i$ and $a[i-1, j-w_i] + v_i \geq a[i-1,j]$

$a[i,j] = a[i-1, j-w_i] + v_i$

(other orderings can also work)