Lecture 5 Dynamic Programming (cont'd)

Tuesday, September 12, 2017 2:33 PM

1. Correctness Proof for Knapsack

Proof by induction.

say $(i,j) < (i',j')$ if $i < i'$ or $(i = i'$ and $j < j')$

$$(0,0) < (0,1) < (0,2) < \cdots < (1,0) < (1,1) < \cdots$$

(IH) induction hypothesis: alg is correct for all values of $a[i,j]$ where $(i,j) < (i',j')$

(all previous elements in table are correct)

base case: $a[i,0] = a[0,j] = 0$ for all i, j

induction step: when computing $a[i',j']$, by IH

$a[i'-1, j']$, $a[i'-1, j'-W_{i'}]$ are already computed <u>correctly</u>

alg considers the optimal value for item i' in Knapsack

$a[i'-1, j'-W_{i'}] + V_{i'}$

for item i' not in Knapsack

$a[i'-1, j']$

$\Rightarrow$ value at $a[i',j']$ is also correct. $\Box$

- Longest Common Subsequence (LCS)

$LCS(a[n], b[m])$

- Last Decision: whether $a[n]$ / $b[m]$ should be in the LCS

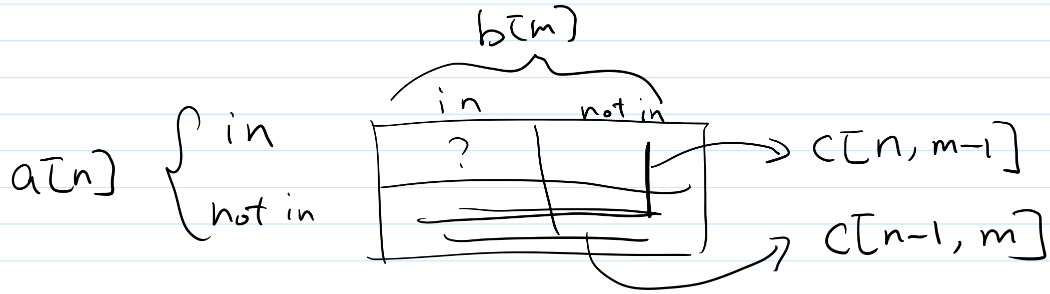
'ababcd$\boxed{e}$' 'abbec$\boxed{d}$'

4 possible cases

$b[m]$

		in	not in
$a[n]$	in		
	not in		

Let $c[i,j]$ be the length of LCS of $a[1..i]$, $b[1...j]$

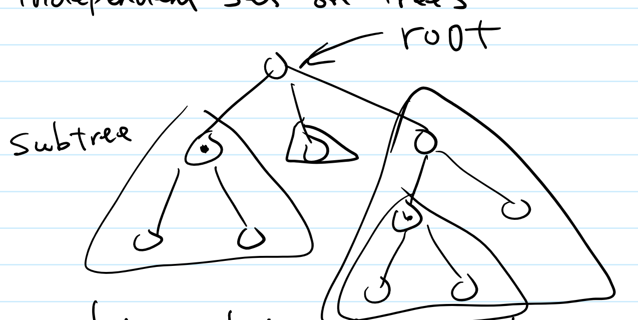


$$c[n,m] = \max \begin{cases} \underline{c[n-1, m]} & \text{case 1} \quad a[n] \text{ not in LCS} \\ \underline{c[n, m-1]} & \text{case 2} \quad b[m] \text{ not in LCS} \\ \underline{c[n-1, m-1]} + 1 & \text{case 3} \quad \text{if } a[n] = b[m] \\ & \qquad a[n], b[m] \text{ both in LCS} \end{cases}$$

base case: if $i=0$, $j=0$ $c[i,j]=0$

ordering: $i = 1$ to n
$j = 1$ to m

- Maximum Independent Set on Trees



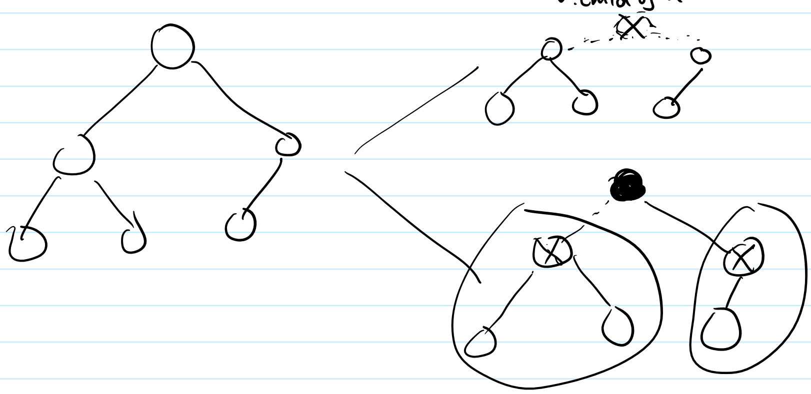
- Goal: relate solution of the whole tree to solutions of the subtrees.
- for the root $\begin{cases} \text{not in the independent set} & \text{(take max indep. set for all children's subtrees)} \\ \text{in the indep. set.} & \end{cases}$

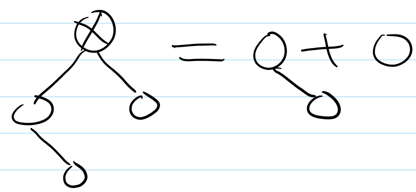
(max indep set on a subtree if root of the subtree cannot be chosen)

$F(u)$ = max ind. set of subtree rooted at u

$G(u)$ = ———————————— but u cannot be in the set

$$F(u) = \max \begin{cases} \sum_{v:\text{child of } u} F(v) & (\text{if } u \text{ is not in set}) \\ \sum_{v:\text{child of } u} G(v) + 1 & (\text{if } u \text{ is in the set}) \end{cases}$$





$$G(u) = \sum_{v:\text{child of } u} F(v) \quad (\text{same as case 1 for } F)$$