| CompSci 516 |
| :---: | :---: |
| Database Systems |
| Lecture 5 |
| Design Theory and |
| Normalization |
| Instructor: Sudeepa Roy |
|  |

## Where are we now?

We learnt
$\checkmark$ Relational Model and Query
Languages
$\checkmark$ SQL, RA, RC
$\checkmark$ Postgres (DBMS)
$\checkmark$ XML (overview)

- HW1

Next

- Database Normalization - (for good schema design)


## Reading Material

- Database normalization
- [RG] Chapter 19.1 to 19.5, 19.6.1, 19.8 (overview) - [GUW] Chapter 3


## What will we learn?

- What goes wrong if we have redundant info in a database?
- Why and how should you refine a schema?
- Functional Dependencies - a new kind of integrity constraints (IC)
- Normal Forms
- How to obtain those normal forms

Duke CS, Fall 2018

| Example |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ssn (S) | name ( N ) | lot <br> (L) | rating (R) | hourlywage (W) | hoursworked (H) |
| 111-11-1111 | Attishoo | 48 | 8 | 10 | 40 |
| 222-22-2222 | Smiley | 22 | 8 | 10 | 30 |
| 333-33-3333 | Smethurst | 35 | 5 | 7 | 30 |
| 444-44-4444 | Guldu | 35 | 5 | 7 | 32 |
| 555-55-5555 | Madayan | 35 | 8 | 10 | 40 |
| - key = SSN <br> - Suppose for a given rating, there is only one hourly_wage value <br> - Redundancy in the table <br> - Why is redundancy bad? |  |  |  |  |  |
| Duke CS, Fall 2018 |  |  |  |  |  |

## Why is redundancy bad?

The list of hourly employees in an organization

| ssn (S) | name (N) | lot <br> $(\mathrm{L})$ | rating <br> $(\mathrm{R})$ | hourly- <br> wage (W) | hours- <br> worked (H) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 111-11-1111 | Attishoo | 48 | 8 | 10 | 40 |
| 222-22-2222 | Smiley | 22 | 8 | 10 | 30 |
| 333-33-3333 | Smethurst | 35 | 5 | 7 | 30 |
| $444-44-4444$ | Guldu | 35 | 5 | 7 | 32 |
| 555-55-5555 | Madayan | 35 | 8 | 10 | 40 |

- Suppose for a given rating, there is only one hourly_wage value
- Redundancy in the table
uke CS, Fall 2018


## Why is redundancy bad?

The list of hourly employees in an organization

| ssn (S) | name (N) | lot <br> (L) | rating <br> (R) | hourly- <br> wage (W) | hours- <br> worked (H) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 111-11-1111 | Attishoo | 48 | 8 | $10 \rightarrow 9$ | 40 |
| 222-22-2222 | Smiley | 22 | 8 | 10 | 30 |
| 333-33-3333 | Smethurst | 35 | 5 | 7 | 30 |
| $444-44-4444$ | Guldu | 35 | 5 | 7 | 32 |
| 555-55-5555 | Madayan | 35 | 8 | 10 | 40 |

## Why is redundancy bad?

The list of hourly employees in an organization

| ssn (S) | name (N) | lot <br> $(\mathrm{L})$ | rating <br> $(\mathrm{R})$ | hourly- <br> wage (W) | hours- <br> worked (H) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 111-11-1111 | Attishoo | 48 | 8 | 10 | 40 |
| 222-22-2222 | Smiley | 22 | 8 | 10 | 30 |
| 333-33-3333 | Smethurst | 35 | 5 | 7 | 30 |
| $444-44-4444$ | Guldu | 35 | 5 | 7 | 32 |
| $555-55-5555$ | Madayan | 35 | 8 | 10 | 40 |

## Why is redundancy bad?

The list of hourly employees in an organization

| ssn (S) | name (N) | lot <br> $($ L) | rating <br> $(\mathbf{R})$ | hourly- <br> wage (W) | hours- <br> worked (H) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 111-11-1111 | Attishoo | 48 | 8 | 10 | 40 |
| 222-22-2222 | Smiley | 22 | 8 | 10 | 30 |
| 333-33-3333 | Smethurst | 35 | 5 | 7 | 30 |
| $444-44-4444$ | Guldu | 35 | 5 | 7 | 32 |
| 555-55-5555 | Madayan | 35 | 8 | 10 | 40 |


| ssn (S) | name (N) | lot <br> $($ L) | rating <br> $($ R) | hourly- <br> wage (W) | hours- <br> worked (H) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 111-11-1111 | Attishoo | 48 | 8 | 10 | 40 |
| 222-22-2222 | Smiley | 22 | 8 | 10 | 30 |
| 333-33-3333 | Smethurst | 35 | 5 | 7 | 30 |
| $444-44-4444$ | Guldu | 35 | 5 | 7 | 32 |
| 555-55-5555 | Madayan | 35 | 8 | 10 | 40 |

## Summary: Redundancy

- Solution?
- decomposition of schema


## Decompositions should be used judiciously

1. Do we need to decompose a relation?

- Several normal forms
- If a relation is not in one of them, may need to decompose further

2. What are the problems with decomposition?

- Lossless joins (soon)
- Performance issues -- decomposition may both
- help performance (for updates, some queries accessing part of data), or
- hurt performance (new joins may be needed for some queries)


## Functional Dependencies (FDs)

- A functional dependency (FD) $X \rightarrow Y$ holds over relation $R$ if, for every allowable instance $r$ of $R$ :
- i.e., given two tuples in $r$, if the $X$ values agree, then the $Y$ values must also agree
- $X$ and $Y$ are sets of attributes
- $t 1 \in r, t 2 \in r, \quad \Pi_{X}(t 1)=\Pi_{X}(t 2)$ implies $\Pi_{Y}(t 1)=\Pi_{Y}(t 2)$

| A | B | C | D |
| :--- | :--- | :--- | :--- |
| a1 | b1 | c1 | d1 |
| a1 | b1 | c1 | d2 |
| a1 | b2 | c2 | d1 |
| a2 | b1 | c3 | d1 |

What is an FD here?
$A B \rightarrow C$
Note that, $A B$ is not a key
not a correct question though.. see next slide!
Duke CS, Fall 2018
Compsci 156: Database Systems

Decomposition

| ssn (S) | name (N) | lot <br> $(\mathrm{L})$ | rating <br> $(\mathrm{R})$ | hourly- <br> wage (W) | hours- <br> worked (H) |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 111-11-1111 | Attishoo | 48 | 8 | 10 | 40 |
| 222-22-2222 | Smiley | 22 | 8 | 10 | 30 |
| 333-33-3333 | Smethurst | 35 | 5 | 7 | 30 |
| $444-44-4444$ | Guldu | 35 | 5 | 7 | 32 |
| $555-55-5555$ | Madayan | 35 | 8 | 10 | 40 |

ssn (S) name (N) lot rating hours-
(L) (R) worked (H)

Attishoo 48 11-1111 8

444-44-4444 Guldu $35 \quad 5$
555-55-5555 Madayan $35 \quad 8 \quad 40$

## Functional Dependencies (FDs)

- A functional dependency (FD) $X \rightarrow Y$ holds over relation $R$ if, for every allowable instance $r$ of R :
- i.e., given two tuples in $r$, if the $X$ values agree, then the $Y$ values must also agree
- X and Y are sets of attributes
- $t 1 \in r, t 2 \in r, \quad \Pi_{X}(t 1)=\Pi_{X}(t 2)$ implies $\Pi_{Y}(t 1)=\Pi_{Y}(t 2)$

| A | B | C | D |
| :--- | :--- | :--- | :--- |
| a1 | b1 | c1 | d1 |
| a1 | b1 | c1 | d2 |
| a1 | b2 | c2 | d1 |
| a2 | b1 | c3 | d1 |

## Functional Dependencies (FDs)

- An FD is a statement about all allowable relations
- Must be identified based on semantics of application
- Given some allowable instance $r 1$ of R, we can check if it violates some $\operatorname{FD} f$, but we cannot tell if $f$ holds over R
- $K$ is a candidate key for $R$ means that $K \rightarrow R$
- denoting $R=$ all attributes of $R$ too
- However, $S \rightarrow R$ does not require $S$ to be minimal
- e.g. S can be a superkey

Duke CS, Fall 2018

## Example

- Consider relation obtained from Hourly_Emps:
- Hourly_Emps (ssn, name, lot, rating, hourly_wage, hours_worked)
- Notation: We will denote a relation schema by listing the attributes: SNLRWH
- Basically the set of attributes $\{S, N, L, R, W, H\}$
- here first letter of each attribute
- FDs on Hourly_Emps:
- ssn is the key: $S \rightarrow$ SNLRWH
- rating determines hourly_wages: $R \rightarrow W$


## Armstrong's Axioms

- $X, Y, Z$ are sets of attributes
- Reflexivity: If $\mathrm{X} \supseteq \mathrm{Y}$, then $\mathrm{X} \rightarrow \mathrm{Y}$
- Augmentation: If $X \rightarrow Y$, then $X Z \rightarrow Y Z$ for any $Z$
- Transitivity: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$
- These are sound and complete inference rules for FDs
- sound: then only generate FDs in $\mathrm{F}^{+}$for F
- complete: by repeated application of these rules, all FDs in $\mathrm{F}^{+}$ will be generated

Duke C5, Fall 2018
Compsci 156: Database Systems

## Armstrong's Axioms

- $X, Y, Z$ are sets of attributes
- Reflexivity: If $X \supseteq Y$, then $X \rightarrow Y$
- Augmentation: If $X \rightarrow Y$, then $X Z \rightarrow Y Z$ for any $Z$
- Transitivity: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$

| A | B | C | D |
| :--- | :--- | :--- | :--- |
| a1 | b1 | c1 | d1 |
| a1 | b1 | c1 | d2 |
| a1 | b2 | c2 | d1 |
| a2 | b1 | c3 | d1 |

Apply these rules on $A B \rightarrow C$ and check

[^0]
## Additional Rules

- Follow from Armstrong's Axioms
- Union: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow Y Z$
- Decomposition: If $X \rightarrow Y Z$, then $X \rightarrow Y$ and $X \rightarrow Z$

| A | B | C | D |
| :--- | :--- | :--- | :--- |
| a1 | b1 | c1 | d1 |
| a1 | b1 | c1 | d2 |
| a2 | b2 | c2 | d1 |
| a2 | b2 | c2 | d2 |

$$
\begin{gathered}
A \rightarrow B, A \rightarrow C \\
A \rightarrow B C \\
A \rightarrow B C
\end{gathered}
$$

$A \rightarrow B, A \rightarrow C$
Duke CS, Fall 2018
Compsci 516: Database Systems

## Closure of a set of FDs

- Given some FDs, we can usually infer additional FDs: - SSN $\rightarrow$ DEPT, and DEPT $\rightarrow$ LOT implies SSN $\rightarrow$ LOT
- An FD $f$ is implied by a set of FDs $F$ if $f$ holds whenever all FDs in $F$ hold.
- $\mathrm{F}^{+}$
$=$ closure of F is the set of all FDs that are implied by $F$

Duke CS, Fall 2018
Compsci 516: Database Systems

## To check if an FD belongs to a closure

- Computing the closure of a set of FDs can be expensive
- Size of closure can be exponential in \#attributes
- Typically, we just want to check if a given FD $X \rightarrow Y$ is in the closure of a set of FDs $F$
- No need to compute $\mathrm{F}^{+}$

1. Compute attribute closure of $X$ (denoted $\mathrm{X}^{+}$) wrt $F$ :

- Set of all attributes $A$ such that $X \rightarrow A$ is in $F^{+}$

2. Check if Y is in $\mathrm{X}^{+}$

Duke CS, Fall $2018 \quad$ Compsci 516: Database Systems

## Computing Attribute Closure

## Algorithm:

- closure = X
- Repeat until no change
- if there is an FD $U \rightarrow V$ in $F$ such that $U \subseteq$ closure, then closure = closure $\cup \mathrm{V}$
- Does $\mathrm{F}=\{\mathrm{A} \rightarrow \mathrm{B}, \mathrm{B} \rightarrow \mathrm{C}, \mathrm{CD} \rightarrow \mathrm{E}\}$ imply $\mathrm{A} \rightarrow$ E?
- i.e, is $\mathrm{A} \rightarrow \mathrm{E}$ in the closure $\mathrm{F}^{+}$? Equivalently, is E in $\mathrm{A}^{+}$?


## FDs play a role in detecting redundancy

Example

- Consider a relation R with 3 attributes, ABC
- No FDs hold: There is no redundancy here - no decomposition needed
- Given A $\rightarrow$ B: Several tuples could have the same A value, and if so, they'll all have the same $B$ value - redundancy decomposition may be needed if $A$ is not a key
- Intuitive idea
- if there is any non-key dependency, e.g. $A \rightarrow B$, decompose!

Duke C5, Fall 2018

## Normal Forms

- Question: given a schema, how to decide whether any schema refinement is needed at all?
- If a relation is in a certain normal forms, it is known that certain kinds of problems are avoided/minimized
- Helps us decide whether decomposing the relation is something we want to do


## Boyce-Codd Normal Form (BCNF)

- Relation $R$ with FDs $F$ is in BCNF if, for all $X \rightarrow$ $A$ in $F$
- A $\in X$ (called a trivial FD), or
- $X$ contains a key for $R$
- i.e. X is a superkey

Next lecture: BCNF decomposition algorithm

Compsci 1516 : Database Systems

Decomposition



## Lossless join decomposition

- Decompose relation $R$ into relations $S$ and $T$
$-\operatorname{attrs}(R)=\operatorname{attrs}(S) \cup \operatorname{attrs}(T)$
$-S=\pi_{\operatorname{attrs}(S)}(R)$
- $T=\pi_{\operatorname{attrs}(T)}(R)$
- The decomposition is a lossless join decomposition if, given known constraints such as FD's, we can guarantee that $R=$ $S \bowtie T$
- $R \subseteq S \bowtie T$ or $R \supseteq S \bowtie T$ ?
- Any decomposition gives $R \subseteq S \bowtie T$ (why?)
- A lossy decomposition is one with $R \subset S \bowtie T$

Duke CS, Fall 2018
CompSci 516: Database Systems

Bad decomposition


- Association between gid and fromDate is lost
- Join returns more rows than the original relation


## Loss? But I got more rows!

- "Loss" refers not to the loss of tuples, but to the loss of information
- Or, the ability to distinguish different original relations



## BCNF decomposition algorithm

- Find a BCNF violation
- That is, a non-trivial FD $X \rightarrow Y$ in $R$ where $X$ is not a super key of $R$
- Decompose $R$ into $R_{1}$ and $R_{2}$, where
- $R_{1}$ has attributes $X \cup Y$
$-R_{2}$ has attributes $X \cup Z$, where $Z$ contains all attributes of $R$ that are in neither $X$ nor $Y$
- Repeat until all relations are in BCNF
- Also gives a lossless decomposition!

CompSci 516: Database System

## BCNF decomposition example - 1

- CSJDPQV, key $\mathrm{C}, \mathrm{F}=\{\mathrm{JP} \rightarrow \mathrm{C}, \mathrm{SD} \rightarrow \mathrm{P}, \mathrm{J} \rightarrow \mathrm{S}\}$
- To deal with SD $\rightarrow$ P, decompose into SDP, CSJDQV.
- To deal with J $\rightarrow$ S, decompose CSJDQV into J S and CJDQV
- Is JP $\rightarrow$ C a violation of BCNF?
- Note:
- several dependencies may cause violation of BCNF
- The order in which we pick them may lead to very different sets of relations
- there may be multiple correct decompositions (can pick J $\rightarrow$ S first Ouke CS, Fall $2018 \quad$ Compsci 516 : Database Systems

BCNF decomposition example - 2


UserJoinsGroup (uid, uname, twitterid, gid, fromDate) BCNF violation: uid $\rightarrow$ uname, twitterid


User (uid, uname, twitterid)
uid $\rightarrow$ uname, twitterid twitterid $\rightarrow$ uid

BCNF


Member (uid, gid, fromDate) uid, gid $\rightarrow$ fromDate BCNF

CompSci 516: Database Systems

## Recap

- Functional dependencies: a generalization of the key concept
- Non-key functional dependencies: a source of redundancy
- BCNF decomposition: a method for removing redundancies
- BCNF decomposition is a lossless join decomposition
- BCNF: schema in this normal form has no redundancy due to FD's

Duke CS, Fall 2018

## Multivalued dependencies

- A multivalued dependency (MVD) has the form
$X \rightarrow Y$, where $X$ and $Y$ are sets of attributes in a relation $R$
- $X \rightarrow Y$ means that whenever two rows in $R$ agree on all the attributes of $X$, then we can swap their $Y$ components and get two rows that are also in $R$



## $B C N F=$ no redundancy?

- User (uid, gid, place)
- A user can belong to multiple groups
- A user can register places she's visited
- Groups and places have nothing to do with other
- FD's?
- None
- BCNF?
- Yes
- Redundancies?
- Tons!

| uid |
| :--- |
| 142 |
| 142 |
| 456 |
| 456 |
| 45 |
| 45 |

Duke CS, Fall 2018
Compsci 156: Database Systems

| uid | gid | place |
| :--- | :--- | :--- |
| 142 | dps | Springfield |
| 142 | dps | Australia |
| 456 | abc | Springfield |
| 456 | abc | Morocco |
| 456 | gov | Springfield |
| 456 | gov | Morocco |

uid $\rightarrow$ uname, twitterid twitterid $\rightarrow$ uid uid, gid $\rightarrow$ fromDate
UserJoinsGroup (uid, uname, twitterid, gid, fromDate)


BCNF UserJoinsGroup' (twitterid, uname, gid, fromDdte) twitterid $\rightarrow$ uname twitterid, gid $\rightarrow$ fromDate
BCNF violation: twitterid $\rightarrow$ uname

UserName (twitterid, uname) Member (twitterid, gid, fromDate) BCNF

BCNF
Duke C5, Fall 2018
MVD examples
User (uid, gid, place)

- uid $\rightarrow$ gid
- uid $\rightarrow$ place
- Intuition: given uid, attributes gid and place are
"independent"
- uid, gid $\rightarrow$ place
- Trivial: LHS $\cup$ RHS = all attributes of $R$
- uid, gid $\rightarrow$ uid
- Trivial: LHS $\supseteq$ RHS
oute cs, sall2018


## Complete MVD + FD rules

- FD reflexivity, augmentation, and transitivity
- MVD complementation:

If $X \rightarrow Y$, then $X \rightarrow \operatorname{attrs}(R)-X-Y$

- MVD augmentation:

If $X \rightarrow Y$ and $V \subseteq W$, then $X W \rightarrow Y V$

- MVD transitivity:

If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z-Y$

- Replication (FD is MVD):

If $X \rightarrow Y$, then $X \rightarrow Y$

- Coalescence:

Try proving things using these!?
If $X \rightarrow Y$ and $Z \subseteq Y$ and there is some $W$ disjoint from $Y$ such that $W \rightarrow Z$, then $X \rightarrow Z$

Duke C5, Fall 2018

## Proof by chase

- In $R(A, B, C, D)$, does $A \rightarrow B$ and $B \rightarrow C$ imply that $A \rightarrow C$ ?

$$
\begin{aligned}
& \begin{array}{l|l|l|l|l|}
\text { Have: } & \boldsymbol{A} & \boldsymbol{B} & \boldsymbol{C} & \boldsymbol{D} \\
\hline
\end{array} \\
& \begin{array}{llll}
a & b_{1} & c_{1} & d_{1}
\end{array} \\
& \text { Need: } \begin{array}{ll|l|l|l}
\boldsymbol{A} & \boldsymbol{B} & \boldsymbol{C} & \boldsymbol{D}
\end{array} \\
& a \quad b_{1} c_{2} d_{1} \\
& \text { a } b_{2} c_{2} d_{2} \\
& a b_{2} c_{1} d_{2} \\
& A \rightarrow B \quad \begin{array}{lllll}
a & b_{2} & c_{1} & d_{1}
\end{array} \\
& B \rightarrow C \quad a \quad b_{2} c_{1} d_{2} \\
& a \quad b_{2} c_{2} d_{1} \\
& B \rightarrow C \begin{array}{l|l|l|l}
a & b_{1} & c_{2} & d_{1} \\
& a & b_{1} & c_{1} \\
\hline
\end{array}
\end{aligned}
$$

## Read this slide after looking at the examples <br> An elegant solution: "chase"

- Given a set of FD's and MVD's $\mathcal{D}$, does another dependency $d$ (FD or MVD) follow from $\mathcal{D}$ ?
- Procedure
- Start with the premise of $d$, and treat them as "seed" tuples in a relation
- Apply the given dependencies in $\mathcal{D}$ repeatedly
- If we apply an FD, we infer equality of two symbols
- If we apply an MVD, we infer more tuples
- If we infer the conclusion of $d$, we have a proof
- Otherwise, if nothing more can be inferred, we have a counterexample


## Another proof by chase

- In $R(A, B, C, D)$, does $A \rightarrow B$ and $B \rightarrow C$ imply that $A \rightarrow C$ ?

$$
\begin{aligned}
& \begin{array}{ll|l|l|l}
\text { Have: } & \boldsymbol{A} & \boldsymbol{B} & \boldsymbol{C} & \boldsymbol{D}
\end{array} \\
& \begin{array}{llll}
a & b_{1} & c_{1} & d_{1}
\end{array} \\
& \text { Need } \\
& c_{1}=c_{2} \\
& \begin{array}{llll}
a & b_{2} & c_{2} & d_{2}
\end{array} \\
& A \rightarrow B \quad b_{1}=b_{2} \\
& B \rightarrow C \quad c_{1}=c_{2}
\end{aligned}
$$

In general, with both MVD's and FD's, chase can generate both new tuples and new equalities

## 4NF

- A relation $R$ is in Fourth Normal Form (4NF) if
- For every non-trivial MVD $X \rightarrow Y$ in $R, X$ is a superkey
- That is, all FD's and MVD's follow from "key $\rightarrow$ other attributes" (i.e., no MVD's and no FD's besides key functional dependencies)
- 4NF is stronger than BCNF
- Because every FD is also a MVD


## 4NF decomposition algorithm

- Find a 4NF violation
- A non-trivial MVD $X \rightarrow Y$ in $R$ where $X$ is not a superkey
- Decompose $R$ into $R_{1}$ and $R_{2}$, where
- $R_{1}$ has attributes $X \cup Y$
- $R_{2}$ has attributes $X \cup Z$ (where $Z$ contains $R$ attributes not in $X$ or $Y$ )
- Repeat until all relations are in 4NF
- Almost identical to BCNF decomposition algorithm
- Any decomposition on a 4NF violation is lossless

Puke CS, Fall 2018

## Other kinds of dependencies and normal forms

- Dependency preserving decompositions
- Join dependencies
- Inclusion dependencies
- 5NF, 3NF, 2NF
- See book if interested (not covered in class)


## 4NF decomposition example



Data should depend on the key, the whole key,
and nothing but the key!

- You could have multiple keys though
- Redundancy is not desired typically
- not always, mainly due to performance reasons
- Functional/multivalued dependencies - capture redundancy
- Decompositions - eliminate dependencies
- Normal forms
- Guarantees certain non-redundancy
- BCNF, and 4NF
- Lossless join
- How to decompose into BCNF, 4NF
- Chase
- Chas.

He
e CS, Fall $2018 \quad$ Compsci 516: Database System


## Summary

- Philosophy behind BCNF, 4NF


[^0]:    Uuke C, fall 2018

