1 CSPs with Binary Constraints

Consider a CSP with finite variable domains, where constraints are represented as tables.

1. Show how a ternary constraint can be turned into three binary constraints by using an auxiliary variable. Next, show how \( k \)-ary constraints can be treated similarly. Finally, show how unary constraints can be eliminated. This demonstrates that any CSP can be transformed into a CSP with only binary constraints.

2. Given a CSP with \( n \) variables over a domain of size \( d \) and \( m \) \( k \)-ary constraints, using the method above to convert it to a binary CSP, how big will the resulting CSP be in the worst case (in terms of number of variables, domain size, and number of constraints)? How much space is required to store the new constraint tables in the worst case?

2 \( k \)-CNF SAT

Describe how to convert a 4-CNF SAT problem into an equivalent 3-CNF SAT problem. Does this work going from 3SAT to 2SAT? Why?

3 2-CNF SAT

1. Consider using CSP variable elimination to solve 2-CNF SAT instances. If we do this the naive way, how large will the domain of the first new variable be in the worst case?

2. Consider using resolution to solve the same problem. Can you provide an upper bound on the total number of new clauses you will generate? What does this imply about the complexity of solving 2-CNF SAT instances?

3. When can you modify the variable elimination procedure to add new edges (constraints) instead of new variables? Does this fix the problem in part (1)?

4 Computer Network

Consider a computer network represented as an undirected graph (nodes are computers and edges are links between them). Each link is half-duplex; it can be used for transmission in either direction, but not simultaneously in both directions. Each machine has a number of ports equal to the number of links attached to that machine. Each port can be used either as an output (O), or as an input (I), or remain idle (D). Each computer can configure its ports arbitrarily, under the following constraints:
At most one port can be used as an input.

If there is an input port, a non-empty subset of the other ports must be used as outputs.

If there is no input, there must be no outputs.

If a port is configured as input, the port at the other end of the link must be an output.

If a port is configured as output, the port at the other end of the link must be an input.

If a port is idle, the port at the other end of the link must be idle, too.

Suppose that we want to configure the network so that we can transmit messages from some computer (source) to a (non-empty) subset of the other machines (targets). For the source and the targets we have the following additional constraints:

- The source must set some non-empty subset of ports as outputs and must have no inputs.
- The targets must have one input and may, or may not, have outputs.

The figure above shows a simple network of four computers (A,B,C,D). Each port is labelled with two letters; the first is the name of the machine it belongs to, and the second is the name of the machine it connects to. Assuming that A is the source and (C,D) are the targets, here are some valid port configurations:

- \((AB=\text{O}, AC=\text{O}, BA=\text{I}, BC=\text{O}, CA=\text{I}, CB=\text{I}, CD=\text{O}, DC=\text{I})\)
- \((AB=\text{O}, AC=\text{D}, BA=\text{I}, BC=\text{O}, CA=\text{D}, CB=\text{I}, CD=\text{O}, DC=\text{I})\)

The following configuration would not be valid (C has two inputs):

- \((AB=\text{O}, AC=\text{O}, BA=\text{I}, BC=\text{O}, CA=\text{I}, CB=\text{I}, CD=\text{O}, DC=\text{I})\)

1. We want to describe the problem as a CSP using one variable for each port. What is the domain of each variable? How many constraints apply on each variable? Specify all the constraints (as tables) for a port of a computer that has a total of 3 ports. Consider separately the cases of a source, a target, or other. You may use the example above, but try to be as generic as possible.

2. What is the total number of possible valid port configurations of a computer with 5 ports? Assume that the machine is neither a source, nor a target.

3. Is a 2-consistency algorithm sufficient to solve this problem? Justify your answer.

5 Consistency

Consider a CSP with binary constraints such that the longest simple cycle in the corresponding constraint graph is of length \(k\). Show that if the CSP is strongly \(k\)-consistent, then it is globally consistent.