Combining Turing Machines

We will define notation that will make it easier to look at more complicated Turing machines.

1. Given Turing Machines $M_1$ and $M_2$
   
   Notation for
   
   - Run $M_1$
   - Run $M_2$

   
   
   2. Given Turing Machines $M_1$ and $M_2$
   
   Notation for
   
   - Run $M_1$
   - If $x$ is current symbol
     - then Run $M_2$
3. Given Turing Machines M1, M2, and M3

Notation for

- Run M1
- If x is current symbol
  - then Run M2
  - else Run M3

More Notation for Simplifying Turing Machines

Suppose $\Gamma = \{a, b, c, B\}$

- z is any symbol in $\Gamma$
- x is a specific symbol from $\Gamma$

1. s - start
2. R - move right
3. L - move left
4. x - write x (and don’t move)
5. $R_a$ - move right until you see an $a$

6. $L_a$ - move left until you see an $a$

7. $R_{\sim a}$ - move right until you see anything that is not an $a$

8. $L_{\sim a}$ - move left until you see anything that is not an $a$

9. $h$ - halt in a final state

10. $\begin{align*}
    a & \xrightarrow{b} w
\end{align*}$

    If the current symbol is $a$ or $b$, let $w$ represent the current symbol.
Example

Assume input string $w \in \Sigma^+$, $\Sigma = \{a, b\}$.

If $|w|$ is odd, then write a $b$ at the end of the string. The tape head should finish pointing at the leftmost symbol of $w$.

input: bab, output: babb

input: ba, output: ba

What is the running time?
Example

Assume input string \( w \in \Sigma^+ \), \( \Sigma = \{a, b\} \), \(|w| > 0\)

For each \( a \) in the string, append a \( b \) to the end of the string.

input: \( abbabb \), output: \( abbabbb \)

The tape head should finish pointing at the leftmost symbol of \( w \).

Turing’s Thesis Any computation that can be carried out by a mechanical means can be performed by a TM.

Definition: An algorithm for a function \( f: D \rightarrow R \) is a TM \( M \), which given input \( d \in D \), halts with answer \( f(d) \in R \).

Example: \( f(x + y) = x + y \), \( x \) and \( y \) unary numbers.

\[
\begin{align*}
\text{start with:} & \quad 111+1111 \\
\uparrow & \\
\text{end with:} & \quad 111111 \\
\uparrow & 
\end{align*}
\]
**Example:** Copy a String, \( f(w) = w0w \), \( w \in \Sigma^* \), \( \Sigma = \{a, b, c\} \)

Denoted by \( C \)

start with: \( \text{abac} \)

\[ \uparrow \]

end with: \( \text{abac0abac} \)

\[ \uparrow \]

**Algorithm:**

- Write a 0 at end of string
- For each symbol in string
  - make a copy of the symbol
**Example:** Shift the string that is to the left of the tape head to the right, denoted by $S_R$ (shift right)

Below, “ba” is to the left of the tape head, so shift “ba” to the right.

```
start with: aaBbabca
     ↑
end with:   aaBBbaca
     ↑
```

Algorithm:

- remember symbol to the right and erase it
- for each symbol to the left do
  - shift the symbol one cell to the right
- replace first symbol erased
- move tape head to appropriate position

```
s R \overset{\mathrm{a,b,c,B}}{\longrightarrow} v \quad 0
     ↓
L B L \overset{\mathrm{a,b,c}}{\longrightarrow} w \to B R w L
     ↓
   B
   ↓
R 0 v L h
```
**Example:** Shift the string that is to the right of tape head to the left, denote by $S_L$ (shift left)

start with: $babcaBba$

$\uparrow$

end with: $bacaBBba$

$\uparrow$

(similar to $S_R$)

\[
\begin{align*}
  &s \quad L \xrightarrow{a,b,c,B} \quad v \rightarrow \quad 0 \\
  &R \quad B \quad R \xrightarrow{a,b,c} \quad w \rightarrow \quad B \quad L \quad w \quad R \\
  &L \quad 0 \quad v \quad R \quad h
\end{align*}
\]
Example: Add unary numbers

This time use shift.

Example: Multiply two unary numbers, \( f(x \cdot y) = x \cdot y \), \( x \) and \( y \) unary numbers. Assume \( x, y > 0 \).

start with: \[ 1111 \star 11 \]

\[ \uparrow \]

end with: \[ 11111111 \]

\[ \uparrow \]