Section: Turing Machines - Building Blocks

1. Given Turing Machines M1 and M2

Notation for

- Run M1
- Run M2

\[
\begin{align*}
&\text{M1} \\
&S \quad H \\
\rightarrow
\end{align*}
\]

\[
\begin{align*}
&\text{M2} \\
&S' \quad H' \\
\rightarrow
\end{align*}
\]

\[
\begin{align*}
&\rightarrow \text{M1} \rightarrow \text{M2}
\end{align*}
\]

\[
\begin{align*}
&S \quad H \\
&z;z,R \\
&S\end{align*}
\]

\[
\begin{align*}
&z;z,L \\
&H' \quad H' \\
\end{align*}
\]

\[
\begin{align*}
&z \text{ represents any symbol in }
\end{align*}
\]
2. Given Turing Machines M1 and M2

M1

M2

\[ \rightarrow M1 \xrightarrow{x} M2 \]

\[ \rightarrow S \quad \overset{x;\text{x,R}}{\longrightarrow} \quad H \quad \overset{z;\text{z,L}}{\longrightarrow} \quad S' \quad \overset{z}{\longrightarrow} \quad H' \]

z represents any symbol in
x is an element of
3. Given Turing Machines $M_1$, $M_2$, and $M_3$

$x$ is an element of $\Gamma$
$y$ is any element except $x$ from $\Gamma$
$z$ is any element from $\Gamma$
More Notation for Simplifying Turing Machines

Suppose $\Gamma = \{a, b, c, B\}$

- $z$ is any symbol in $\Gamma$
- $x$ is a specific symbol from $\Gamma$

1. $s$ - start
2. $R$ - move right
3. $L$ - move left
4. $x$ - write $x$ (and don’t move)
5. $R_a$ - move right until you see an $a$
6. $L_a$ - move left until you see an $a$

7. $R_{\neg a}$ - move right until you see anything that is not an $a$

8. $L_{\neg a}$ - move left until you see anything that is not an $a$

9. $h$ - halt in a final state
10. $\begin{array}{c} a, b \end{array} \Rightarrow \begin{array}{c} w \end{array}$

If the current symbol is $a$ or $b$, let $w$ represent the current symbol.
Example

Assume input string $w \in \Sigma^+$, $\Sigma = \{a, b\}$.

If $|w|$ is odd, then write a $b$ at the end of the string. The tape head should finish pointing at the leftmost symbol of $w$.

input: bab, output: babb
input: ba, output: ba
What is the running time?
Example

Assume input string $w \in \Sigma^+$, $\Sigma = \{a, b\}$, $|w| > 0$

For each $a$ in the string, append a $b$ to the end of the string.

input: $abbabb$, output: $abbabbbb$

The tape head should finish pointing at the leftmost symbol of $w$. 
Turing’s Thesis: Any computation that can be carried out by a mechanical means can be performed by a TM.

Definition: An algorithm for a function $f:D \rightarrow \mathbb{R}$ is a TM $M$, which given input $d \in D$, halts with answer $f(d) \in \mathbb{R}$.

Example: $f(x + y) = x + y$, $x$ and $y$ unary numbers.

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start with: 111+1111
↑
end with: 1111111
↑
```
Example: Copy a String, \( f(w) = w0w, \) 
\( w \in \Sigma^*, \Sigma = \{a, b, c\} \)

Denoted by \( C \)

start with: \( abac \)

\[ \uparrow \]

end with: \( abac0abac \)

\[ \uparrow \]

Algorithm:

- Write a 0 at end of string
- For each symbol in string
  - make a copy of the symbol
Example: Shift the string that is to the left of the tape head to the right, denoted by $S_R$ (shift right)

Below, “ba” is to the left of the tape head, so shift “ba” to the right.

start with: aaBbabc

end with: aaBBbaca
Algorithm:

- remember symbol to the right and erase it
- for each symbol to the left do
  - shift the symbol one cell to the right
- replace first symbol erased
- move tape head to appropriate position
Example: Shift the string that is to the right of tape head to the left, denote by $S_L$ (shift left)

start with: \hspace{1cm} babcaBba

\[\uparrow\]

end with: \hspace{1cm} bacaBBBba

\[\uparrow\]

(similar to $S_R$)
Example: Add unary numbers
This time use shift.

Example: Multiply two unary numbers, \( f(x*y) = x*y \), \( x \) and \( y \) unary numbers. Assume \( x,y > 0 \).

\[
\begin{align*}
\text{start with:} & \quad 1111 \times 11 \\
\uparrow & \\
\text{end with:} & \quad 11111111 \\
\uparrow &
\end{align*}
\]