Why Study Games?

• Many human activities can be modeled as games
  – Negotiations
  – Bidding
  – TCP/IP
  – Military confrontations
  – Pursuit/Evasion
• Games are used to train the mind
  – Human game-playing, animal play-fighting

Why Are Games Good for AI?

• Games typically have concise rules
• Well-defined starting and end points
• Sensing and effecting are simplified
  – Not true for sports games
  – See robocup
• Games are fun!
• Downside: Getting taken seriously (not)
  – See robo search and rescue

History of Games in AI

• Computer games have been around almost as long as computers (perhaps longer)
  – Chess: Turing (and others) in the 1950s
  – Checkers: Samuel, 1950s learning program
• Usually start with naïve optimism
• Follow with naïve pessimism
• Simon: Computer chess champ by 1967
• Many, e.g., Kasparov, predicted that a computer would never be champion

Games Today

• Computers perform at champion level
  – Backgammon, Checkers, Chess, Othello
• Computers perform well
  – Bridge
• Computers still do badly
  – Go, Hex

Game Setup

• Most commonly, we study games that are:
  – 2 player
  – Alternating
  – Zero-sum
  – Perfect information
• Examples: Checkers, chess, backgammon
• Assumptions can be relaxed at some expense
• Economics studies case where number of agents is very large
  – Individual actions don’t change the dynamics
Zero Sum Games

- Assign values to different outcomes
- Win = 1, Loss = -1
- With zero sum games every gain comes at the other player’s expense
- Sum of both player’s scores must be 0
- Are any games truly zero sum?

Characterizing Games

- Two-player games are very much like search
  - Initial state
  - Successor function
  - Terminal test
  - Objective function (heuristic function)
- Unlike search
  - Terminal states are often a large set
  - Full search to terminal states usually impossible

Game Trees

Minimax

- Max player tries to maximize his return
- Min player tries to minimize his return
- This is optimal for both (zero sum)

\[
\text{minimax}(n_{\text{max}}) = \max_{s \in \text{successors}(n)} \text{minimax}(s)
\]

\[
\text{minimax}(n_{\text{min}}) = \min_{s \in \text{successors}(n)} \text{minimax}(s)
\]
Minimax Properties

- Minimax can be run depth first
  - Time $O(b^m)$
  - Space $O(bm)$
- Assumes that opponent plays optimally
- Based on a worst-case analysis
- What if this is incorrect?

Minimax in the Real World

- Search trees are too big
- Alternating turns double depth of the search
  - 2 ply = 1 full turn
- Branching factors are too high
  - Chess: 35
  - Go: 361
- Search from start never terminates in non-trivial games

Evaluation Functions

- Like heuristic functions
- Try to estimate value of a node without expanding all the way to termination
- Using evaluation functions
  - Do a depth-limited search
  - Treat evaluation function as if it were terminal
- What’s wrong with this?
- How do you pick the depth?
- How do you manage your time?
  - Iterative deepening, quiescence

Desiderata for Evaluation Functions

- Would like to put the same ordering on nodes (even if values aren’t totally right)
- Is this a reasonable thing to ask for?
- What if you have a perfect evaluation function?
- How are evaluation functions made in practice?
  - Buckets
  - Linear combinations
    - Chess pieces (material)
    - Board control (positional, strategic)

Search Control Issues

- Horizon effects
  - Sometimes something interesting is just beyond the horizon
  - How do you know?
- When to generate more nodes?
- If you selectively extend your frontier, how you decide where?
- If you have a fixed amount of total game time, how do you allocate this?

Pruning

- The most important search control method is figuring out which nodes you don’t need to expand
- Use the fact that we are doing a worst-case analysis to our advantage
  - Max player cuts off search when he knows min player can force him to make a worse move
  - Min player cuts of search when he knows max player will avoid current subtree
Alpha-beta pruning

Max node pruning

Implementing alpha-beta

Amazing facts about alpha-beta

What About Probabilities?

How to prune

- We still do (bounded) DFS
- Expand at least one path to the “bottom”
- If current node is max node, and min can force a lower value, then prune siblings
- If current node is min node, and max can force a higher value, then prune siblings

max_value(state, alpha, beta)
if cutoff(state) then return eval(state)
for each s in successors(state) do
alpha = max(alpha, min_value(s, alpha, beta))
if alpha >= beta the return beta
end
return alpha

min_value(state, alpha, beta)
if cutoff(state) then return eval(state)
for each s in successors(state) do
beta = min(alpha, max_value(s, alpha, beta))
if beta <= alpha the return alpha
end
return beta

• Empirically, alpha-beta has the effect of reducing the branching factor by half for many problems
• This effectively doubles the horizon that can be searched
• Alpha-beta makes the difference between novice and expert computer players
Expectiminimax

- $n$ random outcomes per chance node
- $O(b^mn^m)$ time

\[
\text{eminimax } (n_{\text{max}}) = \max_{s \in \text{successors}(n)} \text{eminimax } (s) \\
\text{eminimax } (n_{\text{min}}) = \min_{s \in \text{successors}(n)} \text{eminimax } (s) \\
\text{eminimax } (n_{\text{chance}}) = \sum_{s \in \text{successors}(n)} \text{eminimax } (s)p(s)
\]

Expectiminimax is nasty

- High branching factor
- Randomness makes evaluation functions difficult
  - Hard to predict many steps into future
  - Values tend to smear together
  - Preserving order is not sufficient
- Pruning is problematic
  - Need to prune based upon bound on an expectation
  - Need a priori bounds on the evaluation function

Multiplier Games

- Things sort-of generalize
- We can maintain a vector of possible values for each player at each node
- Assume that each player acts greedily
- What’s wrong with this?

Conclusions

- Game tree search is a special kind of search
- Rely heavily on heuristic evaluation functions
- Alpha-beta is a big win
- Most successful players use alpha-beta
- Final thought: Tradeoff between search effort and evaluation function effort
- When is it better to invest in your evaluation function?