Online Association Rule Mining

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Background

- Mining for association rules is a form of data mining
- An example:
  - 65% of all customers who buy pasta and tomato sauce also buy parmesan cheese and red wine
- Useful for customer segmentation, cross-marketing, catalog design and product placement
- Online aggregation

Association Rule

- An association rule is an expression X => Y where X and Y are disjoint itemsets
  - The confidence of this rule is the fraction of all transactions containing X that also contain Y
  - The support of this rule is the support of X U Y.
- A support must >= a user-specified support threshold
- A confidence must >= a user-specified confidence threshold

What’s the problem?

- Finding association rules is a very CPU and memory intensive task
- All traditional algorithms operate offline
- User does not know the appropriate thresholds in advance

Traditional algorithms

- Apriori
- Partition
- Dynamic Itemset Counting
- OLAP-style
  - Support threshold s specified before the precomputation of the large itemsets
  - Large Itemset computation remains offline
  - Only rules with support >= s can be generated

Carma

- Continuous feedback
  - continuously produces association rules, while the list of purchases is scanned
- user controllable
  - During the first scan the user is free to change the support and confidence thresholds "on the fly"
- deterministic and accurate results
  - guarantees that it produces all association rules after at most 2 scans and for each rule its precise support and confidence value
Carma Algorithm: Phase I

- Count(v): number of occurrence of itemset v since v was inserted
- First Trans(v): index of the transaction at which v was inserted
- maxMissd(v): upper bound on the number of occurrences of v before v was inserted
- Minsupport: \( \frac{\text{count}(v)}{i} \) lower bound
- Maxsupport: \( \frac{(\text{maxMissed}(v) + \text{count}(v))}{i} \) upper bound

Carma Algorithm: Phase I

- Support lattice: a superset of all large itemsets
- Support sequence: a sequence of support thresholds \( \sigma = (\sigma_1, \sigma_2, \ldots) \) denotes support threshold for the i-th transaction
- \( \lceil \sigma \rceil \) denotes the least monotone decreasing sequence. It is called ceiling of \( \sigma \) up to i. A sharp lower bound relative to which V is a support lattice.
- \( \text{Avg}(\sigma) = \frac{1}{\sum_{j=1}^{i} \sigma_j} \) (where \( 1 \leq j \leq i \)) It's the running average of \( \sigma \) up to i.

Carma Algorithm: Phase I

- Heart of the algorithm: maxMissed(v)
- maxMissed(v) <= maxMissed(w) + count(w) - 1
  - Support i (w) >= support i (v) for all subsets w of v and w in t
- maxMissed(v) <= \( \sum_{i=1}^{i-1} \text{avg } (\lceil \sigma \rceil) + |v| - 1 \)
  - Support (i-1) (v) <= \( \sum_{i=1}^{i-1} \text{avg } (\lceil \sigma \rceil) + \sum_{v \subset v} \frac{|v| - 1}{i-1} \)
- maxMissed(v) is defined as \( \min \{ \sum_{i=1}^{i-1} \text{avg } (\lceil \sigma \rceil) + \sum_{v \subset v} \frac{|v| - 1}{i-1}, \maxMissed(w) + \text{count}(w) - 1 \} \).
- maxMissed(v) <= i - 1 for the current transaction index.

Carma: theorem 1

- The term \( (c+1)/n \) in desirable
- The term \( \text{avg } (\lceil \sigma \rceil) \) is a sharp lower bound relative to which V is a support lattice
- Support guarantee may not match the threshold specified by the user, but this guarantees *converges* to the user-specified threshold if the user keeps it constant for a large number of transactions

Carma: Phase I algorithm example

- Count(v): number of occurrence of itemset v since v was inserted
- First Trans(v): index of the transaction at which v was inserted
- maxMissd(v): upper bound on the number of occurrences of v before v was inserted
- Minsupport: \( \frac{\text{count}(v)}{i} \) lower bound
- Maxsupport: \( \frac{(\text{maxMissed}(v) + \text{count}(v))}{i} \) upper bound
- maxMissed(v) = \( \min \{ \sum_{i=1}^{i-1} \text{avg } (\lceil \sigma \rceil) + \sum_{v \subset v} \frac{|v| - 1}{i-1}, \maxMissed(w) + \text{count}(w) - 1 \} \).
Carma: Changing support thresholds

- To improve the speed of convergence
  - Run phase I with a lower threshold of $s \times 0.9$ instead of $s$.
  - Increase the threshold from $s \times 0.9$ to $s$, as the guaranteed threshold reaches $s$.

Carma: phase II

Performance: Carma, Apriori and DIC

- At thresholds 0.25% and below Carma outperform Apriori and DIC
  - Less number of scans
  - Smaller lattice maintained by Carma

Carma Implementation

- Dataset with 100k transactions of an average size of 10 items chosen from 10k items and an average large itemset size of 4
- All itemsets are stored in a single hashtable
  - Itemsets as keys: quickly access any subset

Support Intervals

- Phase I maintains a superset of the large itemsets but not necessarily for the full transaction sequence
- Size of the support intervals (given by minsupport and maxsupport)
  - Average size 0.042% at threshold of 0.1% while 50% of all itemsets with an interval size below 0.004%

Conclusion

- Carma compute large itemsets online
- Continuously produces large itemsets along with a shrinking support interval for each itemset
- Allow user to change the support threshold anytime during the first scan and always completes in at most 2 scan
- Carma’s itemset lattice quickly approximates a superset of all large itemset while the sizes of the corresponding support intervals shrink rapidly
- Second scan is not needed when shrinking support intervals suffice so phase I can be used continuously