On homework, you may discuss with other students in the course about how to solve a problem, but the write-up should be your own. You must include the names of any students you consulted with. Give credit where credit is due.

1. (8 pts) Define exchange\( (a_1a_2 \ldots a_{n-1}a_n) = a_n a_2 \ldots a_{n-1} a_1 \) (swap first and last character) and exchange\( (L) = \{ v \mid v = \text{exchange}(w) \text{ for some } w \in L \} \). Show that the family of regular languages is closed under exchange.

For example, if \( abbcc \) is in \( L \), then \( cbbca \) is in \( \text{exchange}(L) \).

2. (12 pts) Prove the following languages are not regular by using the pumping lemma. Show all steps.

(a) \( L = \left\{ a^n b^k \mid n > 0, n \leq k \right\} \)

(b) \( L = \left\{ a^n b^p c^k \mid n > p + k, n > 0, p > 0, k > 0 \right\} \)

(c) \( \Sigma = \{ a, b, c \}, \quad L = \left\{ w \in \Sigma^* \mid n_a(w) \neq n_b(w) \right\}, \) where \( n_x(w) \) means the number of times \( x \) appears in \( w \).

3. (8 pts) Prove that the following languages are NOT regular by using closure properties. Show all steps.

(a) \( L = \left\{ a^n b^p c^k \mid n = p + k, n > 0, p > 0, k > 0 \right\} \)

(b) \( \Sigma = \{ a, b, c \}, \quad L = \left\{ w \in \Sigma^* \mid n_a(w) \leq n_c(w), 0 \leq n_b(w) < n_a(w) + n_c(w) \right\}, \)

where \( n_x(w) \) means the number of times \( x \) appears in \( w \).

4. (9 pts) Write a CFG for each of the following languages.

(a) \( L = \left\{ a^n b^p c^m \mid n > p + m, p > 0, m \geq 0 \right\} \)

(b) \( L = \left\{ a^n b^p c^m \mid n = p + m, p > 0, m \geq 0 \right\} \)

(c) \( L = \left\{ a^n b^p c^m \mid p = n + m, p > 0, m \geq 0 \right\} \)