CPS 140 Homework 7 Spring 2003

Due: Thursday, March 27
48 points

On homework, you may discuss with other students in the course about how to solve a problem, but the write-up should be your own. You must include the names of any students you consulted with. Give credit where credit is due.

This homework is to be written and turned in at the beginning of class. You can use JFLAP to check your answers for some of these, but you still have to turn in the written answer.

1. (8 pts) Consider the following grammar (DO NOT change the grammar):

   \[ S \rightarrow AbC \mid d \]
   \[ A \rightarrow aA \mid \lambda \]
   \[ C \rightarrow Ac \]

   For this problem you will construct the LL(1) parse table.

   (a) Calculate FIRST and FOLLOW for the variables in the grammar.

<table>
<thead>
<tr>
<th></th>
<th>FIRST</th>
<th>FOLLOW</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td></td>
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<tr>
<td>A</td>
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<tr>
<td>C</td>
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</tbody>
</table>

   (b) Calculate all entries in the LL(1) Parse Table. If there are multiple rules for an entry, write in all the rules.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
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</thead>
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</table>

   1
2. (8 pts) Consider the following grammar (DO NOT change the grammar):

\[
S \rightarrow aABd \\
A \rightarrow aA \mid Bc \\
B \rightarrow bBa \mid \lambda
\]

For this problem you will construct the LL(1) parse table.

(a) Calculate FIRST and FOLLOW for the variables in the grammar.

<table>
<thead>
<tr>
<th></th>
<th>FIRST</th>
<th>FOLLOW</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>{a}</td>
<td>{b, c, d, }</td>
</tr>
<tr>
<td>A</td>
<td>{a}</td>
<td>{c}</td>
</tr>
<tr>
<td>B</td>
<td>{b}</td>
<td>|</td>
</tr>
</tbody>
</table>

(b) Calculate all entries in the LL(1) Parse Table. If there are multiple rules for an entry, write in all the rules.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>$</th>
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</thead>
<tbody>
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<td>S</td>
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<td>A</td>
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<td>B</td>
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</tbody>
</table>

3. (3 pts) Consider the following grammar.

\[
S \rightarrow bbC \mid bbc \\
C \rightarrow cC \mid c
\]

The grammar is LL(k) for what value of k?
4. (3 pts) Eliminate useless productions from the following grammar. Use the algorithm from the lecture notes.

\[
\begin{align*}
S & \rightarrow a \mid aA \mid B \mid C \\
A & \rightarrow aB \mid \lambda \\
B & \rightarrow Aa \mid C \\
C & \rightarrow cCD \\
D & \rightarrow ddd
\end{align*}
\]

In particular, give the set \( V_1 \), the dependency graph and the new grammar.

5. (3 pts) Eliminate all \( \lambda \)-productions from the following grammar. Use the algorithm from the lecture notes.

\[
\begin{align*}
S & \rightarrow AaB \mid aaB \\
A & \rightarrow \lambda \\
B & \rightarrow bbA \mid \lambda
\end{align*}
\]

In particular, give the set \( V_N \) and the new grammar.

6. (3 pts) Remove the unit productions from the following grammar. Use the algorithm from the lecture notes.

\[
\begin{align*}
S & \rightarrow a \mid aA \mid B \mid C \\
A & \rightarrow aB \mid \lambda \\
B & \rightarrow Aa \mid C \\
C & \rightarrow cCD \\
D & \rightarrow ddd
\end{align*}
\]

In particular, show the dependency graph, and the new grammar.

7. (3 pts) Convert the following grammar into Chomsky Normal Form.

\[
\begin{align*}
S & \rightarrow abAB \\
A & \rightarrow bAB \mid \lambda \\
B & \rightarrow BAa \mid A \mid \lambda
\end{align*}
\]

8. (15 pts) Using the pumping lemma, prove the following languages are not context-free languages.

(a) \( L = \{a^nb^pc^p \mid 0 < p < n\} \)

(b) \( L = \{a^nb^qc^q \mid n > p > q > 0\} \)

(c) \( L = \{ w \in \Sigma^* \mid n(a) > n(b), n(b) = n(c), \Sigma = \{a, b, c\}, n(a) = \text{number of a's in } w \} \) For example, abcacbaa \( \in L \).