On homework, you may discuss with other students in the course about how to solve a problem, but the write-up should be your own. You must include the names of any students you consulted with. Give credit where credit is due.

This homework is to be turned in at the beginning of class.

1. (24 pts) Using the building block notation for TM (such as $R_a$ which means move right until you see an $a$ (moves first then checks for the $a$)). You may also use the notation of the machines we created in class such as $S_L$ for shift left and $C$ for copy.

Also note that you can include the $|$ symbol (or). $R_{ab}$ means move right until you see an $a$ or a $b$.

For each TM write the building block code and give the big-Oh running time.

(a) $f(x\#y) = T$ if $x < y$ and is equal to $F$ otherwise, where $x$ and $y$ are unary numbers.

For example, $f(111\#11)$ writes $F$ to the tape, and $f(11\#1111)$ writes $T$ to the tape. The answer must be surrounded by blanks. It is ok to have other stuff still on the tape when the Turing machine halts.

Let $|x| = n$ and $|y| = m$. What is the worst case running time (big-Oh) of your Turing machine?

(b) $f(w) = a^n b^m$ where $w \in \Sigma^*$, $\Sigma = \{a, b\}$.

For example, $f(abbaa)$ writes $aaabb$ to the tape, and $f(bbbaa)$ writes $aabb$ to the tape. The answer must be surrounded by blanks. It is ok to have other stuff still on the tape when the Turing machine halts.

Note $|w| = n + m$. What is the worst case running time (big-Oh) of your Turing machine?

(c) Multiply two unary numbers, $f(x\#y) = x\#y$, $x$ and $y$ unary numbers. Assume $x, y > 0$.

\[
\begin{array}{c}
\text{start with:} \\
1111\#11 \\
\uparrow
\end{array}
\]

\[
\begin{array}{c}
\text{end with:} \\
11111111 \\
\uparrow
\end{array}
\]

Let $|x| = n$ and $|y| = m$. What is the worst case running time (big-Oh) of your Turing machine?