Combining Turing Machines

We will define notation that will make it easier to look at more complicated Turing machines

1. Given Turing Machines M1 and M2
   Notation for
   - Run M1
   - Run M2

2. Given Turing Machines M1 and M2
   Notation for
   - Run M1
   - If \( x \) is current symbol
     - then Run M2
3. Given Turing Machines M1, M2, and M3

Notation for

- Run M1
- If x is current symbol
  - then Run M2
  - else Run M3

\[
\begin{align*}
M1 & \xrightarrow{x} H \\
M2 & \xrightarrow{x} H'
n\text{SH} \quad M2
n\text{H'S'}
M3 & \xrightarrow{x} H''
\end{align*}
\]

More Notation for Simplifying Turing Machines

Suppose \( \Gamma = \{a, b, c, B\} \)

- z is any symbol in \( \Gamma \)
- x is a specific symbol from \( \Gamma \)

1. s - start
2. R - move right

3. L - move left

4. x - write x (and don’t move)
5. R_a - move right until you see an a

6. L_a - move left until you see an a

7. R_{\sim a} - move right until you see anything that is not an a

8. L_{\sim a} - move left until you see anything that is not an a

9. h - halt in a final state

10. $a,b \xrightarrow{w}$

   If the current symbol is a or b, let w represent the current symbol.
Example

Assume input string $w \in \Sigma^+$, $\Sigma = \{a, b\}$.

If $|w|$ is odd, then write a $b$ at the end of the string. The tape head should finish pointing at the leftmost symbol of $w$.

input: bab, output: babb

input: ba, output: ba

What is the running time?
Example
Assume input string $w \in \Sigma^+, \Sigma = \{a, b\}, |w| > 0$

For each $a$ in the string, append a $b$ to the end of the string.

input: $abbabb$, output: $abbabbbb$

The tape head should finish pointing at the leftmost symbol of $w$.

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Turing’s Thesis
Any computation that can be carried out by a mechanical means can be performed by a TM.

Definition: An algorithm for a function $f: \mathbb{D} \rightarrow \mathbb{R}$ is a TM $M$, which given input $d \in \mathbb{D}$, halts with answer $f(d) \in \mathbb{R}$.

Example: $f(x + y) = x + y$, $x$ and $y$ unary numbers.

\[
\begin{array}{c}
\text{start with: } \\
111 + 1111 \\
\uparrow \\
\text{end with: } \\
1111111 \\
\uparrow \\
\end{array}
\]
**Example:** Copy a String, $f(w) = w0w$, $w \in \Sigma^*$, $\Sigma = \{a, b, c\}$

Denoted by $C$

- start with: $abac$
  - $\uparrow$
- end with: $abac0abac$
  - $\uparrow$

**Algorithm:**

- Write a 0 at end of string
- For each symbol in string
  - make a copy of the symbol
**Example:** Shift the string that is to the left of the tape head to the right, denoted by $S_R$ (shift right)

Below, “ba” is to the left of the tape head, so shift “ba” to the right.

<table>
<thead>
<tr>
<th>start with:</th>
<th>aaBbabca</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>↑</td>
</tr>
<tr>
<td>end with:</td>
<td>aaBBbaca</td>
</tr>
<tr>
<td></td>
<td>↑</td>
</tr>
</tbody>
</table>

Algorithm:

- remember symbol to the right and erase it
- for each symbol to the left do
  - shift the symbol one cell to the right
- replace first symbol erased
- move tape head to appropriate position

$$s \quad R \quad \underset{\text{v}}{\text{a,b,c,B}} \quad \arrêt{\text{v}}{0}$$

$$L \quad B \quad L \quad \underset{\text{w}}{\text{a,b,c}} \quad \arrêt{\text{w}}{B \quad R \quad w \quad L}$$

$$R_{0} \quad v \quad L \quad h$$
**Example:** Shift the string that is to the right of tape head to the left, denote by \( S_L \) (shift left)

- start with: \( \text{babcaBba} \)
- end with: \( \text{bacaBBba} \)

(similar to \( S_R \))
Example: Add unary numbers

This time use shift.

Example: Multiply two unary numbers, $f(x\times y) = x \times y$, $x$ and $y$ unary numbers. Assume $x,y > 0$.

\[
\begin{align*}
\text{start with:} & \quad 1111 \times 11 \\
\text{end with:} & \quad 11111111
\end{align*}
\]