Section: Turing Machines - Building Blocks

1. Given Turing Machines M1 and M2

Notation for

- Run M1
- Run M2

\( \text{M1} \quad \text{M2} \)

\[ z;z,R \quad z;z,L \]

\( z \) represents any symbol in
2. Given Turing Machines M1 and M2

M1

\[ \rightarrow S \quad H \]

M2

\[ \rightarrow S' \quad H' \]

\[ \rightarrow M1 \xrightarrow{x} M2 \]

\[ \rightarrow S \quad H \xrightarrow{x;R} z;L \quad S' \quad H' \]

\[ z \text{ represents any symbol in} \]

\[ x \text{ is an element of} \]
3. Given Turing Machines M1, M2, and M3

x is an element of

y is any element except x from

z is any element from
More Notation for Simplifying Turing Machines

Suppose \( \Gamma = \{a,b,c,B\} \)

- \( z \) is any symbol in \( \Gamma \)
- \( x \) is a specific symbol from \( \Gamma \)

1. \( s \) - start
2. \( R \) - move right
3. \( L \) - move left
4. \( x \) - write \( x \) (and don’t move)
5. \( R_a \) - move right until you see an \( a \)
6. \( L_a \) - move left until you see an \( a \)

7. \( R_{\neg a} \) - move right until you see anything that is not an \( a \)

8. \( L_{\neg a} \) - move left until you see anything that is not an \( a \)

9. \( h \) - halt in a final state

10. \( \rightarrow \{ a, b \} \rightarrow w \)

If the current symbol is \( a \) or \( b \), let \( w \) represent the current symbol.
Example

Assume input string $w \in \Sigma^+$, $\Sigma = \{a, b\}$. If $|w|$ is odd, then write a $b$ at the end of the string. The tape head should finish pointing at the leftmost symbol of $w$.

input: bab, output: babb
input: ba, output: ba
What is the running time?
Example

Assume input string $w \in \Sigma^+$, $\Sigma = \{a, b\}$, $|w| > 0$

For each $a$ in the string, append a $b$ to the end of the string.

input: $abbabb$, output: $abbabbbb$

The tape head should finish pointing at the leftmost symbol of $w$.  

Turing’s Thesis Any computation that can be carried out by a mechanical means can be performed by a TM.

Definition: An algorithm for a function \( f: D \rightarrow R \) is a TM \( M \), which given input \( d \in D \), halts with answer \( f(d) \in R \).

Example: \( f(x + y) = x + y \), \( x \) and \( y \) unary numbers.

\[
\begin{align*}
\text{start with:} & \quad 111 + 1111 \\
\uparrow & \\
\text{end with:} & \quad 11111111 \\
\uparrow & 
\end{align*}
\]
Example: Copy a String, $f(w) = w0w$, $w \in \Sigma^*, \Sigma = \{a, b, c\}$

Denoted by $C$

- start with: abac
- end with: abac0abac

Algorithm:

- Write a 0 at end of string
- For each symbol in string
  - make a copy of the symbol
Example: Shift the string that is to the left of the tape head to the right, denoted by $S_R$ (shift right)

Below, “ba” is to the left of the tape head, so shift “ba” to the right.

\[
\text{start with: aaBbabc}\ \\
\uparrow\\
\text{end with: aaBBbaca}
\]
Algorithm:

- remember symbol to the right and erase it
- for each symbol to the left do
  - shift the symbol one cell to the right
- replace first symbol erased
- move tape head to appropriate position
Example: Shift the string that is to the right of tape head to the left, denote by $S_L$ (shift left)

start with: \[
\text{babcaBba}
\]

\[\uparrow\]

end with: \[
\text{bacaBBba}
\]

\[\uparrow\]

(similar to $S_R$)
Example: Add unary numbers
This time use shift.

Example: Multiply two unary numbers, $f(x\times y) = x\times y$, $x$ and $y$ unary numbers. Assume $x, y > 0$.

start with: \[1111 \times 11\]

end with: \[11111111\]