Deterministic Finite Acceptor (or Automata)

A DFA = (Q, Σ, δ, q₀, F)

where
- Q is finite set of states
- Σ is tape (input) alphabet
- q₀ is initial state
- F ⊆ Q is set of final states.
- δ: Q × Σ → Q

Example: Create a DFA that accepts even binary numbers.

Transition Diagram:

M = (Q, Σ, δ, q₀, F) =

Tabular Format

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
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<tbody>
<tr>
<td>q₀</td>
<td>q₁</td>
<td>q₀</td>
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<tr>
<td>q₁</td>
<td>q₁</td>
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Example of a move: δ(q₀, 1) =
Algorithm for DFA:

Start in start state with input on tape
q = current state
s = current symbol on tape
while (s != blank) do
    q = δ(q, s)
    s = next symbol to the right on tape
if q ∈ F then accept

Example of a trace: 11010

Pictorial Example of a trace for 100:

1) \[ \begin{array}{c}
q_0 \\
q_1
\end{array} \]

2) \[ \begin{array}{c}
q_0 \\
q_1
\end{array} \]

3) \[ \begin{array}{c}
1 \ 0 \ 0 \\
q_0 \\
q_1
\end{array} \]

4) \[ \begin{array}{c}
1 \ 0 \ 0 \\
q_0 \\
q_1
\end{array} \]

Definition:

\[ \delta^*(q, \lambda) = q \]

\[ \delta^*(q, wa) = \delta(\delta^*(q, w), a) \]

Definition The language accepted by a DFA M=(Q,Σ,δ,q₀,F) is set of all strings on Σ accepted by M. Formally,

\[ L(M) = \{ w \in \Sigma^* \mid \delta^*(q_0, w) \in F \} \]
**Trap State**

Example: \( L(M) = \{ b^n a \mid n > 0 \} \)

You don’t need to show trap states! Any arc not shown will by default go to a trap state.

**Example:** Create a DFA that accepts even binary numbers that have an even number of 1’s.

**Example:**

\( L = \{ w \in \Sigma^* \mid w \text{ has an even number of a’s and an even number of b’s} \} \)

**Definition** A language is regular iff there exists DFA \( M \) s.t. \( L = L(M) \).
Chapter 2.2

Nondeterministic Finite Automata (or Acceptor)

Definition

An NFA = (Q, Σ, δ, q₀, F)

where

Q is finite set of states
Σ is tape (input) alphabet
q₀ is initial state
F ⊆ Q is set of final states.
δ: Q × (Σ ∪ {λ}) → 2^Q

Example

Note: In this example δ(q₀, a) =

Example

L = \{(ab)^n \mid n > 0\} \cup \{a^n b \mid n > 0\}

Definition q_j ∈ δ^*(q_i, w) if and only if there is a walk from q_i to q_j labeled w.

Example From previous example:

δ^*(q₀, ab) =

δ^*(q₀, aba) =

Definition: For an NFA M, L(M) = \{w ∈ Σ^* \mid δ^*(q₀, w) ∩ F ≠ ∅\}

The language accepted by nfa M is all strings w such that there exists a walk labeled w from the start state to final state.
2.3 NFA vs. DFA: Which is more powerful?

Example:

\[ \begin{array}{c}
q_0 \quad \underset{\text{a}}{\rightarrow} \quad q_1 \\
\text{b} \quad \underset{\text{b}}{\rightarrow} \quad q_1 \\
\text{a} \quad \underset{\text{a}}{\rightarrow} \quad q_2 \\
\end{array} \]

**Theorem** Given an NFA \( M_N = (Q_N, \Sigma, \delta_N, q_0, F_N) \), then there exists a DFA \( M_D = (Q_D, \Sigma, \delta_D, q_0, F_D) \) such that \( L(M_N) = L(M_D) \).

Proof:

We need to define \( M_D \) based on \( M_N \).

\( Q_D = \)  
\( F_D = \)  
\( \delta_D : \)

**Algorithm to construct** \( M_D \)

1. start state is \( \{q_0\} \cup \text{closure}(q_0) \)

2. While can add an edge  
   (a) Choose a state \( A = \{q_i, q_j, \ldots q_k\} \) with missing edge for \( a \in \Sigma \)  
   (b) Compute \( B = \delta^*(q_i, a) \cup \delta^*(q_j, a) \cup \ldots \cup \delta^*(q_k, a) \)  
   (c) Add state \( B \) if it doesn’t exist  
   (d) add edge from \( A \) to \( B \) with label \( a \)

3. Identify final states

4. if \( \lambda \in L(M_N) \) then make the start state final.
Minimizing Number of states in DFA

Why?

Algorithm

- Identify states that are indistinguishable
  These states form a new state

Definition Two states \( p \) and \( q \) are indistinguishable if for all \( w \in \Sigma^* \)

\[
\delta^*(q, w) \in F \Rightarrow \delta^*(p, w) \in F \\
\delta^*(p, w) \notin F \Rightarrow \delta^*(q, w) \notin F
\]

Definition Two states \( p \) and \( q \) are distinguishable if \( \exists w \in \Sigma^* \) s.t.

\[
\delta^*(q, w) \in F \Rightarrow \delta^*(p, w) \notin F \text{ OR} \\
\delta^*(q, w) \notin F \Rightarrow \delta^*(p, w) \in F
\]
Example:
Example: