Section: Finite Automata

Deterministic Finite Accepter (or Automata)

A DFA = (Q, Σ, δ, q₀, F)

where
Q is finite set of states
Σ is tape (input) alphabet
$q_0$ is initial state
F ⊆ Q is set of final states.
$\delta: Q \times \Sigma \rightarrow Q$
Example: DFA that accepts even binary numbers.

Transition Diagram:

\[ M=(Q, \Sigma, \delta, q_0, F) = \]

Tabular Format

<table>
<thead>
<tr>
<th></th>
<th>0</th>
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<tbody>
<tr>
<td>q0</td>
<td>q1</td>
<td>q0</td>
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<tr>
<td>q1</td>
<td>q1</td>
<td>q0</td>
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Example of a move: \( \delta(q_0, 1) = \)
Algorithm for DFA:

Start in start state with input on tape
q = current state
s = current symbol on tape
while (s != blank) do
    q = δ(q,s)
    s = next symbol to the right on tape
if q ∈ F then accept

Example of a trace: 11010
Pictorial Example of a trace for 100:

1)  

\[
\begin{array}{c}
1 \quad 0 \quad 0 \\
\uparrow \\
\text{q0} \\
\text{q1}
\end{array}
\]

2)  

\[
\begin{array}{c}
1 \quad 0 \quad 0 \\
\uparrow \\
\text{q0} \\
\text{q1}
\end{array}
\]

3)  

\[
\begin{array}{c}
1 \quad 0 \quad 0 \\
\uparrow \\
\text{q0} \\
\text{q1}
\end{array}
\]

4)  

\[
\begin{array}{c}
1 \quad 0 \quad 0 \\
\uparrow \\
\text{q0} \\
\text{q1}
\end{array}
\]


Definition:
\[ \delta^*(q, \lambda) = q \]
\[ \delta^*(q, wa) = \delta(\delta^*(q, w), a) \]

Definition The language accepted by a DFA \( M=(Q, \Sigma, \delta, q_0, F) \) is set of all strings on \( \Sigma \) accepted by \( M \). Formally,
\[ L(M) = \{ w \in \Sigma^* \mid \delta^*(q_0, w) \in F \} \]
Trap State

Example: \( L(M) = \{ b^n a \mid n > 0 \} \)

Example: DFA that accepts even binary numbers that have an even number of 1’s.
Example:

\[ L = \{ w \in \Sigma^* \mid w \text{ has an even number of a’s and an even number of b’s} \} \]
Definition A language is regular iff there exists DFA $M$ s.t. $L=L(M)$.

Chapter 2.2
Nondeterministic Finite Automata (or Accepter)
Definition
An NFA $=(Q,\Sigma,\delta,q_0,F)$
where
$Q$ is finite set of states
$\Sigma$ is tape (input) alphabet
$q_0$ is initial state
$F \subseteq Q$ is set of final states.
\[\delta:Q \times (\Sigma \cup \{\lambda\}) \rightarrow 2^Q\]
Note: In this example $\delta(q_0, a) =$

Example

$L=\{(ab)^n \mid n > 0\} \cup \{a^n b \mid n > 0\}$
Definition \( q_j \in \delta^*(q_i, w) \) if and only if there is a walk from \( q_i \) to \( q_j \) labeled \( w \).

Example From previous example:

\[
\delta^*(q_0, ab) =
\]

\[
\delta^*(q_0, aba) =
\]

Definition: For an NFA \( M \),

\[
L(M) = \{ w \in \Sigma^* \mid \delta^*(q_0, w) \cap F \neq \emptyset \}
\]
2.3 NFA vs. DFA: Which is more powerful?

Example:
Theorem Given an NFA
\( M_N = (Q_N, \Sigma, \delta_N, q_0, F_N) \), then there
exists a DFA \( M_D = (Q_D, \Sigma, \delta_D, q_0, F_D) \)
such that \( L(M_N) = L(M_D) \).

Proof:

We need to define \( M_D \) based on \( M_N \).

\( Q_D = \)

\( F_D = \)

\( \delta_D : \)
Algorithm to construct $M_D$

1. start state is $\{q_0\} \cup \text{closure}(q_0)$

2. While can add an edge
   (a) Choose a state $A = \{q_i, q_j, \ldots q_k\}$ with missing edge for $a \in \Sigma$
   (b) Compute $B = \delta^*(q_i, a) \cup \delta^*(q_j, a) \cup \ldots \cup \delta^*(q_k, a)$
   (c) Add state $B$ if it doesn’t exist
   (d) add edge from $A$ to $B$ with label $a$

3. Identify final states

4. if $\lambda \in L(M_N)$ then make the start state final.
Example:
Minimizing Number of states in DFA

Why?

Algorithm

- Identify states that are indistinguishable

These states form a new state

Definition Two states $p$ and $q$ are indistinguishable if for all $w \in \Sigma^*$

$$\delta^*(q, w) \in F \Rightarrow \delta^*(p, w) \in F$$
$$\delta^*(p, w) \notin F \Rightarrow \delta^*(q, w) \notin F$$

Definition Two states $p$ and $q$ are distinguishable if $\exists w \in \Sigma^*$ s.t.

$$\delta^*(q, w) \in F \Rightarrow \delta^*(p, w) \notin F \text{ OR }$$
$$\delta^*(q, w) \notin F \Rightarrow \delta^*(p, w) \in F$$
Example:
Example: