Review

Regular Languages

- FA, RG, RE
- recognize

Context Free Languages

- PDA, CFG
- recognize

**DFA:**

![DFA Diagram]

**Turing Machine:**

![Turing Machine Diagram]

**Turing Machine (TM)**

- invented by Alan M. Turing (1936)
- computational model to study algorithms
Definition of TM

- Storage
  - tape
- actions
  - write symbol
  - read symbol
  - move left (L) or right (R)
- computation
  - initial configuration
    * start state
    * tape head on leftmost tape square
    * input string followed by blanks
  - processing computation
    * move tape head left or right
    * read from and write to tape
  - computation halts
    * final state

Formal Definition of TM

A TM $M$ is defined by $M=(Q,\Sigma,\Gamma,\delta,q_0,B,F)$ where

- $Q$ is finite set of states
- $\Sigma$ is input alphabet
- $\Gamma$ is tape alphabet
- $B\in \Gamma$ is blank
- $q_0$ is start state
- $F$ is set of final states
- $\delta$ is transition function
  $\delta(q,a) = (p,b,R)$ means “if in state $q$ with the tape head pointing to an 'a', then move into state $p$, write a 'b' on the tape and move to the right”.

TM as Language recognizer

Definition: Configuration is denoted by $\vdash$.

if $\delta(q,a) = (p,b,R)$ then a move is denoted

abaqabba $\vdash$ ababpbbba
**Definition:** Let $M$ be a TM, $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$. $L(M) = \{ w \in \Sigma^* | q_0 \xrightarrow{w} x_1 q_f x_2 \text{ for some } q_f \in F, x_1, x_2 \in \Gamma^* \}$

**TM as language acceptor**

$M$ is a TM, $w$ is in $\Sigma^*$,

- if $w \in L(M)$ then $M$ halts in final state
- if $w \notin L(M)$ then either
  - $M$ halts in non-final state
  - $M$ doesn’t halt

**Example**

$\Sigma = \{a, b\}$

Replace every second ’a’ by a ’b’ if string is even length.

- Algorithm
Example:

$L = \{a^n b^n c^n | n \geq 1\}$

Is the following TM correct?

```
2;2,R
a;a,R
b;2,R
3;3,R
b;b,R
1;1,R
2;2,R
3;3,L
```

```
1;1, R
```

```
a;a,L
b;b,L
2;2,L
3;3,L
```

**TM as a transducer**

TM can implement a function: $f(w) = w'$

```
\begin{array}{c}
\text{start with: } w \\
\uparrow \\
\text{end with: } w' \\
\uparrow \\
\end{array}
```

**Definition:** A function with domain $D$ is *Turing-computable* or *computable* if there exists TM $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ such that

$q_0 w \vdash^* q_f f(w)$

$q_f \in F$, for all $w \in D$.

**Example:**

$f(x) = 2x$

$x$ is a unary number

```
\begin{array}{c}
\text{start with: } 111 \\
\uparrow \\
\text{end with: } 111111 \\
\uparrow \\
\end{array}
```
Is the following TM correct?

Example:

$L = \{ww \mid w \in \Sigma^+\}, \Sigma = \{a, b\}$