Why Study Games?

- Many human activities can be modeled as games
  - Negotiations
  - Bidding
  - TCP/IP
  - Military confrontations
  - Pursuit/Evasion
- Games are used to train the mind
  - Human game-playing, animal play-fighting

Why Are Games Good for AI?

- Games typically have concise rules
- Well-defined starting and end points
- Sensing and effecting are simplified
  - Not true for sports games
  - See robocup
- Games are fun!
- Downside: Getting taken seriously (not)
  - See robo search and rescue

History of Games in AI

- Computer games have been around almost as long as computers (perhaps longer)
  - Chess: Turing (and others) in the 1950s
  - Checkers: Samuel, 1950s learning program
- Usually start with naïve optimism
- Follow with naïve pessimism
- Simon: Computer chess champ by 1967
- Many, e.g., Kasparov, predicted that a computer would never be champion
Games Today

- Computers perform at champion level
  - Backgammon, Checkers, Chess, Othello
- Computers perform well
  - Bridge
- Computers still do badly
  - Go, Hex

Game Setup

- Most commonly, we study games that are:
  - 2 player
  - Alternating
  - Zero-sum
  - Perfect information
- Examples: Checkers, chess, backgammon
- Assumptions can be relaxed at some expense
- Economics studies case where number of agents is very large
  - Individual actions don’t change the dynamics

Zero Sum Games

- Assign values to different outcomes
- Win = 1, Loss = -1
- With zero sum games every gain comes at the other player’s expense
- Sum of both player’s scores must be 0
- Are any games truly zero sum?

Characterizing Games

- Two-player games are very much like search
  - Initial state
  - Successor function
  - Terminal test
  - Objective function (heuristic function)
- Unlike search
  - Terminal states are often a large set
  - Full search to terminal states usually impossible
Minimax

- Max player tries to maximize his return
- Min player tries to minimize his return
- This is optimal for both (zero sum)

\[
\text{minimax}(n_{\text{max}}) = \max_{s \in \text{successors}(n)} \text{minimax}(s) \\
\text{minimax}(n_{\text{min}}) = \min_{s \in \text{successors}(n)} \text{minimax}(s)
\]
Minimax Properties

• Minimax can be run depth first
  – Time $O(b^m)$
  – Space $O(bm)$

• Assumes that opponent plays optimally

• Based on a worst-case analysis

• What if this is incorrect?

Minimax in the Real World

• Search trees are too big

• Alternating turns double depth of the search
  – 2 ply = 1 full turn

• Branching factors are too high
  – Chess: 35
  – Go: 361

• Search from start never terminates in non-trivial games

Evaluation Functions

• Like heuristic functions

• Try to estimate value of a node without expanding all the way to termination

• Using evaluation functions
  – Do a depth-limited search
  – Treat evaluation function as if it were terminal

• What’s wrong with this?

• How do you pick the depth?

• How do you manage your time?
  • Iterative deepening, quiescence

Desiderata for Evaluation Functions

• Would like to put the same ordering on nodes (even if values aren’t totally right)

• Is this a reasonable thing to ask for?

• What if you have a perfect evaluation function?

• How are evaluation functions made in practice?
  • Buckets
  • Linear combinations
    • Chess pieces (material)
    • Board control (positional, strategic)
Search Control Issues

• Horizon effects
  – Sometimes something interesting is just beyond the horizon
  – How do you know?
• When to generate more nodes?
• If you selectively extend your frontier, how do you decide where?
• If you have a fixed amount of total game time, how do you allocate this?

Pruning

• The most important search control method is figuring out which nodes you don’t need to expand
• Use the fact that we are doing a worst-case analysis to our advantage
  – Max player cuts off search when he knows min player can force a provably bad outcome
  – Min player cuts of search when he knows max can force a provably good (for max) outcome

Alpha-beta pruning

How to prune

• We still do (bounded) DFS
• Expand at least one path to the “bottom”
• If current node is max node, and min can force a lower value, then prune siblings
• If current node is min node, and max can force a higher value, then prune siblings
Max node pruning

Amazing facts about alpha-beta

- Empirically, alpha-beta has the effect of reducing the branching factor by half for many problems
- This effectively doubles the horizon that can be searched
- Alpha-beta makes the difference between novice and expert computer players

Implementing alpha-beta

```python
max_value(state, alpha, beta)
if cutoff(state) then return eval(state)
for each s in successors(state) do
    alpha = max(alpha, min_value(s, alpha, beta))
    if alpha >= beta then return beta
end
return alpha
```

```python
min_value(state, alpha, beta)
if cutoff(state) then return eval(state)
for each s in successors(state) do
    beta = min(alpha, max_value(s, alpha, beta))
    if beta <= alpha then return alpha
end
return beta
```

What About Probabilities?
**Expectiminimax**

- n random outcomes per chance node
- \(O(b^m n^m)\) time

\[
\begin{align*}
\text{eminimax}(n_{\text{max}}) &= \max_{s \in \text{successors}(n)} \text{eminimax}(s) \\
\text{eminimax}(n_{\text{min}}) &= \min_{s \in \text{successors}(n)} \text{eminimax}(s) \\
\text{eminimax}(n_{\text{chance}}) &= \sum_{s \in \text{successors}(n)} \text{eminimax}(s) p(s)
\end{align*}
\]

**Expectiminimax is nasty**

- High branching factor
- Randomness makes evaluation functions difficult
  - Hard to predict many steps into future
  - Values tend to smear together
  - Preserving order is not sufficient
- Pruning is problematic
  - Need to prune based upon bound on an expectation
  - Need a priori bounds on the evaluation function

**Multiplayer Games**

- Things sort-of generalize
- We can maintain a vector of possible values for each player at each node
- Assume that each player acts greedily
- What’s wrong with this?

**Conclusions**

- Game tree search is a special kind of search
- Rely heavily on heuristic evaluation functions
- Alpha-beta is a big win
- Most successful players use alpha-beta
- Final thought: Tradeoff between search effort and evaluation function effort
- When is it better to invest in your evaluation function?