Please, do not forget to print out, sign, and turn in together with this homework a page with the honor code (e.g., the homework webpage). [CLRS] refers to our course textbook.

Problem 1. Prove that:

(a) $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ for all $n > 0$

(b) $2^n < n!$ for all $n \geq 4$

(c) Assuming that $(1 + \frac{1}{n})^n < e$, for all $n \in \mathbb{N}$ ($\mathbb{N}$ being the set of natural numbers (positive integers)), prove by induction that $n! > (\frac{n}{e})^n$.

Problem 2. Problem 2-4 from [CLRS].

Problem 3. Let $H_n = \sum_{i=1}^{n} \frac{1}{i}$. Prove that $H_n = \Theta(\ln n)$.

Problem 4. Problem 3-3 from [CLRS].

Problem 5. Give asymptotic upper and lower bounds for $T(n)$ in each of the following recurrences. Assume that $T(n)$ is constant for sufficiently small n. Make your bounds as tight as possible, and justify your answers.

(a) $T(n) = 7T(n/3) + n^2$

(b) $T(n) = 7T(n/2) + n^2$

(c) $T(n) = 3T(n/2) + n \lg n$

(d) $T(n) = 3T(n/3 + 5) + n/2$

(e) $T(n) = T(n-1) + 1/n$

(f) $T(n) = \sqrt{n}T(\sqrt{n}) + n$