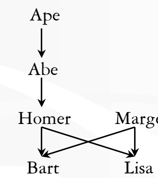
SQL: Recursion

A motivating example

Parent (*parent*, *child*)

parent	*child*
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe



- Example: find Bart's ancestors
- "Ancestor" has a recursive definition
  - *X* is *Y*'s ancestor if
    - *X* is *Y*'s parent, or
    - *X* is *Z*'s ancestor and *Z* is *Y*'s ancestor

Recursion in SQL

- SQL2 had no recursion
  - You can find Bart's parents, grandparents, great grandparents, etc.

```
SELECT p1.parent AS grandparent
FROM Parent p1, Parent p2
WHERE p1.child = p2.parent
AND p2.child = 'Bart';
```

  - But you cannot find all his ancestors with a single query
- SQL3 introduces recursion
  - `WITH` clause
  - Implemented in DB2 (called common table expressions)

Ancestor query in SQL3

```
WITH Ancestor(anc, desc) AS
((SELECT parent, child FROM Parent)
 UNION
 (SELECT a1.anc, a2.desc
  FROM Ancestor a1, Ancestor a2
  WHERE a1.desc = a2.anc))
SELECT anc
FROM Ancestor
WHERE desc = 'Bart';
```

basis

induction

Define a relation recursively

Query using the relation defined in `WITH` clause

How do we compute such a recursive query?

Fixed point of a function

- If f: $T \to T$ is a function from a type T to itself, a fixed point of f is a value x such that $f(x) = x$
- Example: What is the fixed point of $f(x) = x / 2$?
  - 0, because $f(0) = 0 / 2 = 0$
- To compute a fixed point of f
  - Start with a "seed": $x \leftarrow x_0$
  - Compute $f(x)$
    - If $f(x) = x$, stop; x is fixed point of f
    - Otherwise, $x \leftarrow f(x)$; repeat
- Example: compute the fixed point of $f(x) = x / 2$
  - With seed 1: 1, 1/2, 1/4, 1/8, 1/16, … $\to$ 0

Fixed point of a query

- A query q is just a function that maps an input table to an output table, so a fixed point of q is a table T such that $q(T) = T$
- To compute fixed point of q
  - Start with an empty table: $T \leftarrow \varnothing$
  - Evaluate q over T
    - If the result is identical to T, stop; T is a fixed point
    - Otherwise, let T be the new result; repeat

  ☞Starting from $\varnothing$ produces the unique minimal fixed point (assuming q is monotone)

Finding ancestors


```
WITH Ancestor(anc, desc) AS
  ((SELECT parent, child FROM Parent)
   UNION
   (SELECT a1.anc, a2.desc
    FROM Ancestor a1, Ancestor a2
    WHERE a1.desc = a2.anc))
```

Parent (*parent*, *child*)

parent	child
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe

- Think of it as $Ancestor = q(Ancestor)$

anc	desc

→

anc	desc
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe

→

anc	desc
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe
Abe	Bart
Abe	Lisa
Ape	Homer

→

anc	desc
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe
Abe	Bart
Abe	Lisa
Ape	Homer
Ape	Bart
Ape	Lisa

Intuition behind fixed-point iteration

- Initially, we know nothing about ancestor-descendent relationships
- In the first step, we deduce that parents and children form ancestor-descendent relationships
- In each subsequent steps, we use the facts deduced in previous steps to get more ancestor-descendent relationships
- We stop when no new facts can be proven

Mutual recursion example

- Table *Natural* (n) contains 1, 2, …, 100
- Which numbers are even/odd?
  - An odd number plus 1 is an even number
  - An even number plus 1 is an odd number
  - 1 is an odd number

```
WITH Even(n) AS
     (SELECT n FROM Natural
      WHERE n = ANY(SELECT n+1 FROM Odd)),
     Odd(n) AS
     ((SELECT n FROM Natural WHERE n = 1)
      UNION
      (SELECT n FROM Natural
       WHERE n = ANY(SELECT n+1 FROM Even)))
```

Operational semantics of `WITH`

- `WITH` R_1 `AS` Q_1, …,
  R_n `AS` Q_n
  Q;
  - $Q_1, \ldots, Q_n$ may refer to $R_1, \ldots, R_n$
- Operational semantics
  1. $R_1 \leftarrow \varnothing, \ldots, R_n \leftarrow \varnothing$
  2. Evaluate $Q_1, \ldots, Q_n$ using the current contents of $R_1, \ldots, R_n$: $R_1^{\text{new}} \leftarrow Q_1, \ldots, R_n^{\text{new}} \leftarrow Q_n$
  3. If $R_i^{\text{new}} \neq R_i$ for any i
     3.1. $R_1 \leftarrow R_1^{\text{new}}, \ldots, R_n \leftarrow R_n^{\text{new}}$
     3.2. Go to 2.
  4. Compute Q using the current contents of $R_1, \ldots, R_n$ and output the result

Computing mutual recursion


```
WITH Even(n) AS
     (SELECT n FROM Natural
      WHERE n = ANY(SELECT n+1 FROM Odd)),
     Odd(n) AS
     ((SELECT n FROM Natural WHERE n = 1)
      UNION
      (SELECT n FROM Natural
       WHERE n = ANY(SELECT n+1 FROM Even)))
```

- $Even = \varnothing$, $Odd = \varnothing$
- $Even = \varnothing$, $Odd = \{1\}$
- $Even = \{2\}$, $Odd = \{1\}$
- $Even = \{2\}$, $Odd = \{1, 3\}$
- $Even = \{2, 4\}$, $Odd = \{1, 3\}$
- $Even = \{2, 4\}$, $Odd = \{1, 3, 5\}$
- …

Fixed points are not unique


```
WITH Ancestor(anc, desc) AS
  ((SELECT parent, child FROM Parent)
   UNION
   (SELECT a1.anc, a2.desc
    FROM Ancestor a1, Ancestor a2
    WHERE a1.desc = a2.anc))
```

Parent (*parent*, *child*)

parent	child
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe

anc	desc
Homer	Bart
Homer	Lisa
Marge	Bart
Marge	Lisa
Abe	Homer
Ape	Abe
Abe	Bart
Abe	Lisa
Ape	Homer
Ape	Bart
Ape	Lisa
bogus	bogus

Note that the bogus tuple reinforces itself!

- There may be many other fixed points
- But if q is monotone, then all these fixed points must contain the fixed point we computed from fixed-point iteration starting with $\varnothing$
  - Thus the unique minimal fixed point is the "natural" answer to the query

Mixing negation with recursion

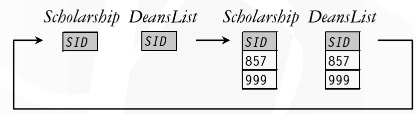
- If q is non-monotone
  - The fixed-point iteration may flip-flop and never converge
  - There could be multiple minimal fixed points—so which one is the right answer?
- Example: reward students with GPA higher than 3.9
  - Those not on the Dean's List should get a scholarship
  - Those without scholarships should be on the Dean's List
  -
```
WITH Scholarship(SID) AS
     (SELECT SID FROM Student WHERE GPA > 3.9
      AND SID NOT IN (SELECT SID FROM DeansList)),
     DeansList(SID) AS
     (SELECT SID FROM Student WHERE GPA > 3.9
      AND SID NOT IN (SELECT SID FROM Scholarship))
```


Fixed-point iteration does not converge

```
WITH Scholarship(SID) AS
     (SELECT SID FROM Student WHERE GPA > 3.9
      AND SID NOT IN (SELECT SID FROM DeansList)),
     DeansList(SID) AS
     (SELECT SID FROM Student WHERE GPA > 3.9
      AND SID NOT IN (SELECT SID FROM Scholarship))
```

Student

SID	name	age	GPA
857	Lisa	8	4.3
999	Jessica	10	4.2

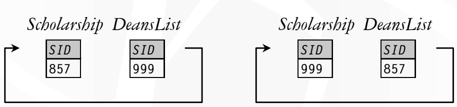


Multiple minimal fixed points

```
WITH Scholarship(SID) AS
     (SELECT SID FROM Student WHERE GPA > 3.9
      AND SID NOT IN (SELECT SID FROM DeansList)),
     DeansList(SID) AS
     (SELECT SID FROM Student WHERE GPA > 3.9
      AND SID NOT IN (SELECT SID FROM Scholarship))
```

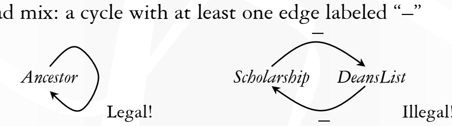
Student

SID	name	age	GPA
857	Lisa	8	4.3
999	Jessica	10	4.2



Legal mix of negation and recursion

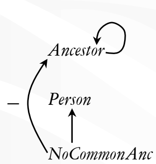
- Construct a dependency graph
  - One node for each table defined in `WITH`
  - A directed edge $R \to S$ if R is defined in terms of S
  - Label the directed edge "–" if the query defining R is not monotone with respect to S
- Legal SQL3 recursion: no cycle containing a "–" edge
  - Called stratified negation
- Bad mix: a cycle with at least one edge labeled "–"



Stratified negation example

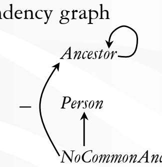
- Find pairs of persons with no common ancestors

```
WITH Ancestor(anc, desc) AS
     ((SELECT parent, child FROM Parent) UNION
      (SELECT a1.anc, a2.desc
       FROM Ancestor a1, Ancestor a2
       WHERE a1.desc = a2.anc)),
     Person(person) AS
     ((SELECT parent FROM Parent) UNION
      (SELECT child FROM Parent)),
     NoCommonAnc(person1, person2) AS
     ((SELECT p1.person, p2.person
       FROM Person p1, Person p2
       WHERE p1.person <> p2.person)
      EXCEPT
      (SELECT a1.desc, a2.desc
       FROM Ancestor a1, Ancestor a2
       WHERE a1.anc = a2.anc))
SELECT * FROM NoCommonAnc;
```



Evaluating stratified negation

- The stratum of a node R is the maximum number of "–" edges on any path from R in the dependency graph
  - *Ancestor*: stratum 0
  - *Person*: stratum 0
  - *NoCommonAnc*: stratum 1



- Evaluation strategy
  - Compute tables lowest-stratum first
  - For each stratum, use fixed-point iteration on all nodes in that stratum
    - Stratum 0: *Ancestor* and *Person*
    - Stratum 1: *NoCommonAnc*

☞Intuitively, there is no negation within each stratum

Summary

- SQL3 `WITH` recursive queries
- Solution to a recursive query (with no negation): unique minimal fixed point
- Computing unique minimal fixed point: fixed-point iteration starting from $\varnothing$
- Mixing negation and recursion is tricky
  - Illegal mix: fixed-point iteration may not converge; there may be multiple minimal fixed points
  - Legal mix: stratified negation (compute by fixed-point iteration stratum by stratum)