

CPS234 COMPUTATIONAL GEOMETRY

Homework # 5

Due date: April 06, 2004, Tuesday (beginning of the class)

Your solutions should be concise, but complete, and typed (or handwritten clearly). Feel free to consult textbooks, journal and conference papers and also each other, but write the solutions yourself and cite your sources. Answer **only four** of the following five questions.

1. **Euler characteristic** [25 pts]. Give an elementary proof for the fact that the Euler characteristic of every triangulation of the 2-dimensional disk is one.
2. **Irreducible sphere** [25 pts]. Give an elementary proof for the claim that the 2-sphere has only one irreducible triangulation, namely the boundary complex of a tetrahedron.
3. **Order complex** [25 pts]. A *flag* in a simplicial complex is a nested sequence of simplices: $\tau_0 < \tau_1 < \dots < \tau_k$, where “ $<$ ” means “proper face of”. Every subsequence of a flag is again a flag. The *order complex* of K is the system of flags in K .
 - (i) Letting K be the boundary complex of a tetrahedron, draw the order complex as a simplicial complex in 3-dimensional space.
 - (ii) Give a general description of the order complex as a simplicial complex.
4. **Minimum square distance** [25 pts]. Let S be a finite set of points in the plane and let f map each point x to the sum of square distances from the points in S : $f(x) = \sum_{p \in S} \|x - p\|^2$.
 - (i) Show that f is a quadratic function that has a unique minimum.
 - (ii) At which point does f attain its minimum?
5. **Points, lines and planes** [25 pts]. Let S be a finite set of points in 3-dimensional space.
 - (i) Is there a finite set of lines, L , in space that generates the same square distance function, $\sum_{p \in S} \|x - p\|^2 = \sum_{\ell \in L} d(x, \ell)^2$?
 - (ii) Similarly, is there a finite set of planes, H , that generates the same square distance function, $\sum_{p \in S} \|x - p\|^2 = \sum_{h \in H} d(x, h)^2$?