These questions will also serve as warmup for recitation 5

1. For the following sequence:

   1, 3, 7, 13, 21, 31, 43

   (a) What is the next value?

   (b) What is the recursion for \(a_n\)?

   (c) What is the closed form for \(a_n\):?

2. Prove that \(\forall n > 6, 3^n < n!\)

3. Prove that a \(2^k\)-by-\(2^k\) chessboard with L-shaped tiles with one corner square is left uncovered.
4. The following is an example where it is easier to prove a stronger result than a weaker one:

(a) Try to prove by induction that \(1 + 1/4 + 1/9 + \cdots + 1/n^2 < 2\) for all positive integers \(n\). What happens?

(b) Prove the same inequality using calculus. [Hint: find an integral whose value is related to that of the series.]

(c) Prove by induction that \(1 + 1/4 + 1/9 + \cdots + 1/n^2 < 2 - 1/n\) for all positive integers \(n\).

5. A married couple organizes a party. They only invite other married couples. At least one person of an invited couple is acquainted to at least the host or the hostess (so between sets host, hostess and male of invited couple, female of invited couple there exists at least one relation, but two, three or four relations is also possible). Upon arrival at the party, each person shakes hands with all other guests he/she doesn’t know yet (it is assumed everybody knows him/herself and his/her partner).

When all couples have arrived and all the handshaking has been done, the host mingles between the guests and ask everybody (including his wife): "How many hands did you shake?". To his surprise, all responses are different.

With the above information, you must be able to determine how many hands the host and hostess each shook.