CPS130 - Homework 3
Due: Thurs, 24 February 2005

1. Let X be a non-negative random variable such that E$[X]$ is well defined. Using this, prove, for all $t > 0$, *Markov's Inequality:*

$$\Pr[X \geq t] \leq \frac{E[X]}{t}$$

2. Consider the Balls and Bins problem with m bins and n balls, with $n \geq m$. Assuming that when balls are thrown into the bins, they're placed uniformly. Now compute (in terms of m and n) the probability that:

   a) There is a bin with *no* balls in it.
   b) At least k of the bins are empty.
   c) There is a bin with less than n/m balls in it.

   Now consider if the probability is *not* uniformly distributed, but rather follows a geometric distribution; that is, the probability of landing in bucket 1 is p, bucket 2 is $(1-p)p$, bucket 3 is $(1-p)^2p$, and so on, until the bucket m is the remainder of the probability (1 - $\sum_{i=0}^{n}(1-p)^i p$).

   d) What is the probability that bucket i is empty?
   e) What is the probability that no bucket is empty?
   f) Compute the expected number of balls in each bucket.

3. Show how we can use two stacks to implement a Queue such that the amortized cost of each *Enqueue* and *Dequeue* is O(1).

4. Show how we can implement a dynamic set that efficiently supports three operations: Enqueue, Dequeue and Min. How can we implement this in:

   a) Enqueue and Dequeue in $O(1)$ time, Min in $O(n)$ time.
   b) Enqueue and Dequeue in $O(\log n)$ time, Min in $O(1)$ time.
   c) All three operations in amortized $O(1)$ time.

Extra Credit Suppose you're given a list of n integers, some negative, some positive. We want to find any three distinct integers from the list so that $a+b+c=0$. How can we do this in $O(n^3)$ time? Descibe how we can reduce this to $O(n^2)$. Can we do better? *This problem is not easy, finish the actual homework problems before trying to do it.*