Section: Finite Automata

Deterministic Finite Acceptor (or Automata)

A DFA = ($Q, \Sigma, \delta, q_0, F$)

where
$Q$ is finite set of states
$\Sigma$ is tape (input) alphabet
$q_0$ is initial state
$F \subseteq Q$ is set of final states.
$\delta : Q \times \Sigma \rightarrow Q$
Example: DFA that accepts even binary numbers.

Transition Diagram:

\[ M = (Q, \Sigma, \delta, q_0, F) = \]

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
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<tbody>
<tr>
<td>( q_0 )</td>
<td>( q_1 )</td>
<td>( q_0 )</td>
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<td>( q_1 )</td>
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Example of a move: \( \delta(q_0, 1) = \)
Algorithm for DFA:

Start in start state with input on tape
q = current state
s = current symbol on tape
while (s != blank) do
  q = δ(q,s)
  s = next symbol to the right on tape
if q∈F then accept

Example of a trace: 11010
Pictorial Example of a trace for 100:

1) 1 0 0

q0
q1

2) 1 0 0

q0
q1

3) 1 0 0

q0
q1

4) 1 0 0

q0
q1
Definition:

\[ \delta^*(q, \lambda) = q \]
\[ \delta^*(q, wa) = \delta(\delta^*(q, w), a) \]

Definition The language accepted by a DFA \( M = (Q, \Sigma, \delta, q_0, F) \) is set of all strings on \( \Sigma \) accepted by \( M \). Formally, \( L(M) = \{ w \in \Sigma^* \mid \delta^*(q_0, w) \in F \} \)
Trap State

Example: $L(M) = \{b^n a \mid n > 0\}$
Example: DFA that accepts even binary numbers that have an even number of 1’s.
Example:

\[ L = \{ w \in \Sigma^* \mid \text{w has an even number of a’s and an even number of b’s} \} \]
Definition A language is regular iff there exists DFA $M$ s.t. $L = L(M)$. 
Chapter 2.2

Nondeterministic Finite Automata (or Accepter)

Definition

An NFA = (Q, Σ, δ, q₀, F)

where

Q is finite set of states
Σ is tape (input) alphabet
q₀ is initial state
F ⊆ Q is set of final states.

δ: Q × (Σ ∪ {λ}) → 2⁰
Example

Note: In this example $\delta(q_0, a) =$
Example

\[ L = \{(ab)^n \mid n > 0\} \cup \{a^n b \mid n > 0\} \]
Definition \( q_j \in \delta^*(q_i, w) \) if and only if there is a walk from \( q_i \) to \( q_j \) labeled \( w \).

Example From previous example:

\[
\delta^*(q_0, ab) =
\]

\[
\delta^*(q_0, aba) =
\]

Definition: For an NFA \( M \),

\[
L(M) = \{ w \in \Sigma^* \mid \delta^*(q_0, w) \cap F \neq \emptyset \}
\]
2.3 NFA vs. DFA: Which is more powerful?

Example:
Theorem Given an NFA $M_N = (Q_N, \Sigma, \delta_N, q_0, F_N)$, then there exists a DFA $M_D = (Q_D, \Sigma, \delta_D, q_0, F_D)$ such that $L(M_N) = L(M_D)$.

Proof:

We need to define $M_D$ based on $M_N$.

$Q_D =$

$F_D =$

$\delta_D :$
Algorithm to construct $M_D$

1. start state is $\{q_0\} \cup \text{closure}(q_0)$

2. While can add an edge
   (a) Choose a state $A=\{q_i, q_j, \ldots q_k\}$ with missing edge for $a \in \Sigma$
   (b) Compute $B = \delta^*(q_i, a) \cup \delta^*(q_j, a) \cup \ldots \cup \delta^*(q_k, a)$
   (c) Add state $B$ if it doesn’t exist
   (d) add edge from $A$ to $B$ with label $a$

3. Identify final states

4. if $\lambda \in L(M_N)$ then make the start state final.
Example:
Minimizing Number of states in DFA

Why?

Algorithm

• Identify states that are indistinguishable
  These states form a new state

Definition Two states p and q are indistinguishable if for all $w \in \Sigma^*$

$$\delta^*(q, w) \in F \Rightarrow \delta^*(p, w) \in F$$
$$\delta^*(p, w) \notin F \Rightarrow \delta^*(q, w) \notin F$$

Definition Two states p and q are distinguishable if $\exists w \in \Sigma^*$ s.t.

$$\delta^*(q, w) \in F \Rightarrow \delta^*(p, w) \notin F \text{ OR } \delta^*(q, w) \notin F \Rightarrow \delta^*(p, w) \in F$$
Example:
Example: