Review

Regular Languages

- FA, RG, RE
- recognize

Context Free Languages

- PDA, CFG
- recognize

DFA:

Turing Machine:

Turing Machine (TM)

- invented by Alan M. Turing (1936)
- computational model to study algorithms
Definition of TM

- **Storage**
  - tape
- **actions**
  - write symbol
  - read symbol
  - move left (L) or right (R)
- **computation**
  - initial configuration
    * start state
    * tape head on leftmost tape square
    * input string followed by blanks
  - processing computation
    * move tape head left or right
    * read from and write to tape
  - computation halts
    * final state

Formal Definition of TM

A TM $M$ is defined by $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ where

- $Q$ is finite set of states
- $\Sigma$ is input alphabet
- $\Gamma$ is tape alphabet
- $B \in \Gamma$ is blank
- $q_0$ is start state
- $F$ is set of final states
- $\delta$ is transition function

$\delta(q,a) = (p,b,R)$ means “if in state $q$ with the tape head pointing to an ‘a’, then move into state $p$, write a ‘b’ on the tape and move to the right”.

TM as Language recognizer

**Definition**: Configuration is denoted by $\vdash$.

If $\delta(q,a) = (p,b,R)$ then a move is denoted

$abaqabba \vdash ababpbba$
**Definition:** Let $M$ be a TM, $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$. $L(M) = \{ w \in \Sigma^* | q_0 \stackrel{*}{\rightarrow} x_1q_fx_2 \text{ for some } q_f \in F, x_1, x_2 \in \Gamma^* \}$

**TM as language acceptor**

$M$ is a TM, $w$ is in $\Sigma^*$,

- if $w \in L(M)$ then $M$ halts in final state
- if $w \notin L(M)$ then either
  - $M$ halts in non-final state
  - $M$ doesn’t halt

**Example**

$\Sigma = \{a, b\}$

Replace every second 'a' by a 'b' if string is even length.

- Algorithm
Example:

$L = \{ a^n b^n c^n | n \geq 1 \}$

Is the following TM correct?

**TM as a transducer**

TM can implement a function: $f(w) = w'$

<table>
<thead>
<tr>
<th>Start with: $w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\uparrow$</td>
</tr>
<tr>
<td>End with: $w'$</td>
</tr>
<tr>
<td>$\uparrow$</td>
</tr>
</tbody>
</table>

**Definition:** A function with domain $D$ is *Turing-computable or computable* if there exists TM $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ such that

$q_0 w \vdash^* q_f f(w)$

$q_f \in F$, for all $w \in D$.

**Example:**

$f(x) = 2x$

$x$ is a unary number

<table>
<thead>
<tr>
<th>Start with: $111$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\uparrow$</td>
</tr>
<tr>
<td>End with: $111111$</td>
</tr>
<tr>
<td>$\uparrow$</td>
</tr>
</tbody>
</table>
Is the following TM correct?

Example:

$L=\{ww \mid w \in \Sigma^+\}$, $\Sigma=\{a, b\}$