Analyzing Algorithms

- Consider three solutions to SortByFreqs, also code used in Anagram assignment
  - Sort, then scan looking for changes
  - Insert into Set, then count each unique string
  - Find unique elements without sorting, sort these, then count each unique string
  - Use a Map (TreeMap or HashMap)

- We want to discuss trade-offs of these solutions
  - Ease to develop, debug, verify
  - Runtime efficiency
  - Vocabulary for discussion

What is big-Oh about? (preview)

- Intuition: avoid details when they don’t matter, and they don’t matter when input size (N) is big enough
  - For polynomials, use only leading term, ignore coefficients
    - $y = 3x$  $y = 6x - 2$  $y = 15x + 44$
    - $y = x^2$  $y = x^2 - 6x + 9$  $y = 3x^2 + 4x$

- The first family is $O(n)$, the second is $O(n^2)$
  - Intuition: family of curves, generally the same shape
  - More formally: $O(f(n))$ is an upper-bound, when $n$ is large enough the expression $c f(n)$ is larger
  - Intuition: linear function: double input, double time, quadratic function: double input, quadruple the time

Recall adding to list (class handout)

- Add one element to front of ArrayList
  - Shift all elements
  - Cost N for N-element list
  - Cost $1 + 2 + \ldots + N = N(N+1)/2$ if repeated

- Add one element to front of LinkedList
  - No shifting, add one link
  - Cost is independent of $N$, constant-time cost
  - Cost $1 + 1 + \ldots + 1 = N$ if repeated

More on O-notation, big-Oh

- Big-Oh hides/obscures some empirical analysis, but is good for general description of algorithm
  - Allows us to compare algorithms in the limit
    - 20N hours vs N^2 microseconds: which is better?
- O-notation is an upper-bound, this means that $N$ is $O(N)$, but it is also $O(N^2)$; we try to provide tight bounds.
  - Formally:
    - A function $g(N)$ is $O(f(N))$ if there exist constants $c$ and $n$ such that $g(N) < cf(N)$ for all $N > n$
Which graph is “best” performance?

Big-Oh calculations from code

- Search for element in an array:
  - What is complexity of code (using O-notation)?
  - What if array doubles, what happens to time?

  ```java
  for(int k=0; k < a.length; k++) {
      if (a[k].equals(target)) return true;
  }
  return false;
  ```

- Complexity if we call N times on M-element vector?
  - What about best case? Average case? Worst case?

Amortization: Expanding ArrayLists

- Expand capacity of list when add() called
- Calling add N times, doubling capacity as needed

<table>
<thead>
<tr>
<th>Item #</th>
<th>Resizing cost</th>
<th>Cumulative</th>
<th>Resizing Cost</th>
<th>Capacity After add</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3-4</td>
<td>4</td>
<td>6</td>
<td>1.5</td>
<td>4</td>
</tr>
<tr>
<td>5-8</td>
<td>8</td>
<td>14</td>
<td>1.75</td>
<td>8</td>
</tr>
<tr>
<td>2^k-1 + 2^k+1</td>
<td>2</td>
<td>2^k-2</td>
<td>around 2</td>
<td>2^k+1</td>
</tr>
</tbody>
</table>

- What if we grow size by one each time?

Some helpful mathematics

- \(1 + 2 + 3 + 4 + \ldots + N\)
  - \(N(N+1)/2\), exactly = \(N^2/2 + N/2\) which is \(O(N^2)\) why?
- \(N + N + N + \ldots + N\) (total of \(N\) times)
  - \(N^2\) which is \(O(N^2)\)
- \(N + N + N + \ldots + N + \ldots + N + \ldots + N\) (total of \(3N\) times)
  - \(3N^2\) which is \(O(N^2)\)
- \(1 + 2 + 4 + \ldots + 2^N\)
  - \(2^{N+1} - 1 = 2 \cdot 2^N - 1\) which is \(O(2^N)\)

- Impact of last statement on adding \(2^N+1\) elements to a vector
  - \(1 + 2 + \ldots + 2^N + 2^{N+1} = 2^{N+2} - 1 = 4 \cdot 2^N - 1\) which is \(O(2^N)\)
### Running times @ $10^6$ instructions/sec

<table>
<thead>
<tr>
<th>N</th>
<th>$O(\log N)$</th>
<th>$O(N)$</th>
<th>$O(N \log N)$</th>
<th>$O(N^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.000003</td>
<td>0.0001</td>
<td>0.000033</td>
<td>0.0001</td>
</tr>
<tr>
<td>100</td>
<td>0.000007</td>
<td>0.0010</td>
<td>0.000664</td>
<td>0.1000</td>
</tr>
<tr>
<td>1,000</td>
<td>0.000010</td>
<td>0.0100</td>
<td>0.010000</td>
<td>1.0</td>
</tr>
<tr>
<td>10,000</td>
<td>0.00013</td>
<td>0.1000</td>
<td>0.132900</td>
<td>1.7 min</td>
</tr>
<tr>
<td>100,000</td>
<td>0.000017</td>
<td>0.1000</td>
<td>1.661000</td>
<td>2.78 hr</td>
</tr>
<tr>
<td>1,000,000</td>
<td>0.000020</td>
<td>1.0</td>
<td>19.9</td>
<td>11.6 day</td>
</tr>
<tr>
<td>1,000,000,000</td>
<td>0.000030</td>
<td>16.7 min</td>
<td>18.3 hr</td>
<td>318 centuries</td>
</tr>
</tbody>
</table>

### Getting in front

- **Suppose we want to add a new element**
  - At the back of a string or an ArrayList or a ...
  - At the front of a string or an ArrayList or a ...
  - Is there a difference? Why? What's complexity?

- **Suppose this is an important problem: we want to grow at the front (and perhaps at the back)**
  - Think editing film clips and film splicing
  - Think DNA and gene splicing

- **Self-referential data structures to the rescue**
  - References, reference problems, recursion, binky

### What’s the Difference Here?

- **How does find-a-track work? Fast forward?**

### Contrast LinkedList and ArrayList

- **See ISimpleList, SimpleLinkedList, SimpleArrayList**
  - Meant to illustrate concepts, not industrial-strength
  - Very similar to industrial-strength, however

- **ArrayList --- why is access $O(1)$ or constant time?**
  - Storage in memory is contiguous, all elements same size
  - Where is the 1st element? 40th? 360th?
  - Doesn’t matter what’s in the ArrayList, everything is a pointer or a reference (what about null?)
What about LinkedList?

- Why is access of Nth element linear time?
- Why is adding to front constant-time O(1)?

ArrayLists and linked lists as ADTs

- As an ADT (abstract data type) ArrayLists support
  - Constant-time or O(1) access to the k-th element
  - Amortized linear or O(n) storage/time with add
    - Total storage used in n-element vector is approx. 2n, spread over all accesses/additions (why?)
    - Adding a new value in the middle of an ArrayList is expensive, linear or O(n) because shifting required
- Linked lists as ADT
  - Constant-time or O(1) insertion/deletion anywhere, but...
  - Linear or O(n) time to find where, sequential search
- Good for sparse structures: when data are scarce, allocate exactly as many list elements as needed, no wasted space/copying (e.g., what happens when vector grows?)

Linked list applications

- Remove element from middle of a collection, maintain order, no shifting. Add an element in the middle, no shifting
  - What’s the problem with a vector (array)?
  - Naively keep characters in a linked list, but in practice too much storage, need more esoteric data structures
- What’s (3x^3 + 2x^2 + x + 5) + (2x^4 + 5x^3 + x^2 + 4x)?
  - As a vector (3, 0, 2, 0, 1, 5) and (0, 2, 5, 1, 4, 0)
  - As a list ((3, 5), (2, 3), (1, 1), (5, 0)) and ________?
- Most polynomial operations sequentially visit terms, don’t need random access, do need “splicing”
- What about (3x^{100} + 5)?

Linked list applications continued

- If programming in C, there are no “growable-arrays”, so typically linked lists used when # elements in a collection varies, isn’t known, can’t be fixed at compile time
  - Could grow array, potentially expensive/wasteful especially if # elements is small.
  - Also need # elements in array, requires extra parameter
  - With linked list, one pointer used to access all the elements in a collection
- Simulation/modelling of DNA gene-splicing
  - Given list of millions of CGTA... for DNA strand, find locations where new DNA/gene can be spliced in
    - Remove target sequence, insert new sequence
Linked lists, CDT and ADT

- **As an ADT**
  - A list is empty, or contains an element and a list
  - ( ) or (x, (y, ( )))

- **As a picture**
  - ![Diagram of a linked list structure]

- **As a CDT (concrete data type)**
  ```java
  public class Node {
    String value;
    Node next;
    Node(String s, Node link) {
      value = s;
      next = link;
    }
  }
  ```

Building linked lists

- **Add words to the front of a list (draw a picture)**
  - Create new node with next pointing to list, reset start of list
  ```java
  public class Node {
    String value;
    Node next;
    Node(String s, Node link) {
      value = s;
      next = link;
    }
  }
  ```

Dissection of add-to-front

- **List initially empty**
- **First node has first word**

- Each new word causes new node to be created
  - New node added to front
- **Rhs of operator = completely evaluated before assignment**

Standard list processing (iterative)

- **Visit all nodes once, e.g., count them or process them**
  ```java
  public int size(Node list) {
    int count = 0;
    while (list != null) {
      count++;
      list = list.next;
    }
    return count;
  }
  ```

- **What changes in code above if we change what “process” means?**
  - Print nodes?
  - Append “s” to all strings in list?
Nancy Leveson: Software Safety

Founded the field
- Mathematical and engineering aspects
  - Air traffic control
  - Microsoft word

"C++ is not state-of-the-art, it’s only state-of-the-practice, which in recent years has been going backwards"

- Software and steam engines: once extremely dangerous?

- THERAC 25: Radiation machine that killed many people

Building linked lists continued

- What about adding a node to the end of the list?
  - Can we search and find the end?
  - If we do this every time, what’s complexity of building an N-node list? Why?

- Alternatively, keep pointers to first and last nodes of list
  - If we add node to end, which pointer changes?
  - What about initially empty list: values of pointers?
    - Will lead to consideration of header node to avoid special cases in writing code

- What about keeping list in order, adding nodes by splicing into list? Issues in writing code? When do we stop searching?

Standard list processing (recursive)

- Visit all nodes once, e.g., count them

```
public int recsize(Node list) {
    if (list == null) return 0;
    return 1 + recsize(list.next);
}
```

- Base case is almost always empty list: null pointer
  - Must return correct value, perform correct action
  - Recursive calls use this value/state to anchor recursion
  - Sometimes one node list also used, two “base” cases

- Recursive calls make progress towards base case
  - Almost always using list.next as argument

Recursion with pictures
Recursion and linked lists

- Print nodes in reverse order
  - Print all but first node and...
    - Print first node before or after other printing?

```
public void print(Node list) {
    if (list != null) {
        System.out.println(list.info);
        print(list.next);
        System.out.println(list.info);
        print(list.next);
    }
}
```

Complexity Practice

- What is complexity of Build? (what does it do?)

```
public Node build(int n) {
    if (null == n) return null;
    Node first = new Node(n, build(n-1));
    for(int k = 0; k < n-1; k++) {
        first = new Node(n,first);
    }
    return first;
}
```

- Write an expression for $T(n)$ and for $T(0)$, solve.
  - Let $T(n)$ be time for build to execute with n-node list
  - $T(n) = T(n-1) + O(n)$

Changing a linked list recursively

- Pass list to method, return altered list, assign to list
  - Idiom for changing value parameters

```
list = change(list, "apple");
public Node change(Node list, String key) {
    if (list != null) {
        list.next = change(list.next, key);
        if (list.info.equals(key)) return list.next;
        else return list;
    }
    return null;
}
```

- What does this code do? How can we reason about it?
  - Empty list, one-node list, two-node list, n-node list
  - Similar to proof by induction