Particle Filters

Ron Parr
CPS 170

Outline

- Problem: Track state over time
  - State = position, orientation of robot (condition of patient, position of airplane, status of factory, etc.)
- Challenge: State is not observed directly
- Solution: Tracking using a model
  - Exact
  - Approximate (Particle filter)

Example

- Robot is monitoring door to the AI lab
- D = variable for status of door (True = open)
- Initially we will ignore observations

- Define Markov model for behavior of door:

$$P(D_{t+1} \mid D_t) = 0.8$$
$$P(D_{t+1} \mid \overline{D}_t) = 0.3$$

Problem

Suppose we believe the door was closed with prob. 0.7 at time t.

What is the prob. that it will be open at time t+1?

$$P(D_{t+1} \mid D_t) = 0.8$$
$$P(D_{t+1} \mid \overline{D}_t) = 0.3$$

Staying open → $P(D_{t+1} \mid D_t)P(D_t)$

Switching from closed to open → $P(D_{t+1} \mid \overline{D}_t)P(\overline{D}_t)$

$$P(D_{t+1}) = P(D_{t+1} \mid D_t)P(D_t) + P(D_{t+1} \mid \overline{D}_t)P(\overline{D}_t)$$
$$= 0.8*0.7 + 0.3*0.3 = 0.65$$

Generalizing

- Suppose states are not binary:

$$P(S_{t+1}) = \sum_{S_t} P(S_{t+1} \mid S_t)P(S_t)$$

- Suppose states are continuous

$$p(S_{t+1}) = \int_{S_t} p(S_{t+1} \mid S_t)p(S_t)dS_t$$

- Issue: For large or continuous states spaces this may be hard to deal with exactly

Monte Carlo Approximation (Sampling)

- We can approximate a nasty integral by sampling and counting:

$$p(S_{t+1}) = \int_{S_t} p(S_{t+1} \mid S_t)p(S_t)dS_t$$

- Repeat n times:
  - Draw sample from $p(S_t)$
  - Simulate transition to S_{t+1}
- Count proportion of states for each value of S_{t+1}

Example

$P(D_{t+1} \mid D_t) = 0.8$
$P(D_{t+1} \mid \overline{D}_t) = 0.3$

- Pick n=1000
  - 700 door open samples
  - 300 door closed samples
- For each sample generate a next state
  - For open samples use prob. 0.8 for next state open
  - For closed samples use prob. 0.3 for next state open
- Count no. of open and closed next states
- Can prove that in limit of large n, our count will equal true probability (0.65)

Example Revisited

- D = Door status
- O = Robot's observation of door status
- Observations may not be completely reliable!

$$P(D_{t+1} \mid D_t) = 0.8$$
$$P(D_{t+1} \mid \overline{D}_t) = 0.3$$
$$P(O \mid D) = 0.6$$
$$P(O \mid \overline{D}) = 0.2$$

Modified Sampling

- Problem: How do we adjust sampling to handle evidence?
- Solution: Weight each sample by the probability of the observations
- Called importance sampling, or likelihood weighting
- Does the right thing for large n

Example with evidence

$P(D_{t+1} \mid D_t) = 0.8$
$P(D_{t+1} \mid \overline{D}_t) = 0.3$
$P(O \mid D) = 0.6$
$P(O \mid \overline{D}) = 0.2$

- Suppose we observe door closed at t+1
- Pick n=1000
  - 700 door open samples
  - 300 door closed samples
- For each sample generate a next state
  - For open samples use prob. 0.8 for next state open
  - For closed samples use prob. 0.3 for next state open
  - If next state is open, weight by 0.4
  - If next state is closed, weight by 0.8
- Compute weighted sum of no. of open and closed states

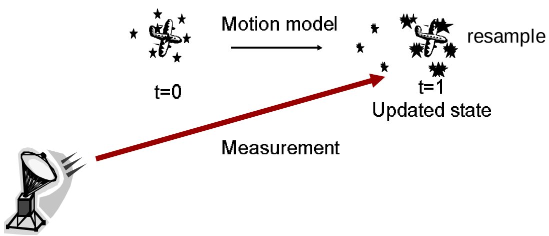
Problems with IS (LW)

- Sequential importance sampling (SIS) does the right thing for the limit of large numbers of samples
- Problems for finite numbers of samples:
  - *Effective* sample size drops over time
  - Unlikely events are only small fraction of sample population
  - Eventually
    - Something unlikely happens
    - A sequence of individually likely events has the effect of a single unlikely event
  - Estimates become unreliable b/c based on a small no. of samples

Solution: SISR (PF)

- Maintain n samples for each time step
- Repeat n times:
  - Draw sample from $p(S_t)$ (according to current weights)
  - Simulate transition to S_{t+1}
  - Weight samples by evidence
- Count proportion of states for each value of S_{t+1}

Monte Carlo Approximation (Particle Filter)



Robot Localization

- Particle filters combine:
  - A model of state change
  - A model of sensor readings
- To track objects with hidden state over time
- Robot application:
  - Hidden state: Robot position, orientation
  - State change model: Robot motion model
  - Sensor model: Laser rangefinder error model
- Note: Robot is tracking itself!

Main Loop

- Sample n robot states
- For each state
  - Simulate next state (action model)
  - Weight states (observation model)
  - Normalize
- Repeat

Robot States

- Robot has X,Y,Z,θ
- Usually ignore z
  - assume floors are flat
  - assume robot stays on one floor
- Form of samples
  - $(X_i, Y_i, \theta_i, p_i)$
  - $\sum_i p_i = 1$

Main Loop

- Sample n robot states
- For each state
  - Simulate next state (action model)
  - Weight states (observation model)
  - Normalize
- Repeat

Sampling Robot States

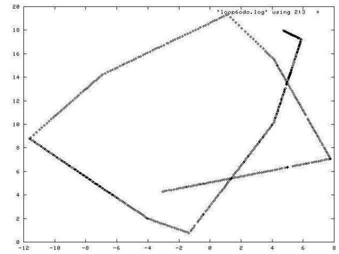
- Need to generate n new samples from our previous set of n samples
- Draw n new robot states with replacement
- for i=1 to n
  - r = rand[0…1]
  - temp = k = 0
  - while(temp <= r)
    - temp=temp+samples[k].p
    - k = k+1
  - newsamples[i] = samples[k-1] (n.b. this should *copy*)
- samples = newsamples

Main Loop

- Sample n robot states
- For each state
  - Simulate next state (action model)
  - Weight states (observation model)
  - Normalize
- Repeat

Action Model

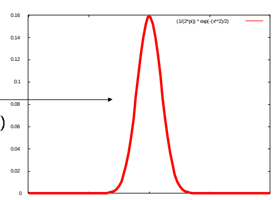
- How far has the robot traveled?
- What does the odometer tell us?



Actual path was a closed loop on the second floor!

Odometer Model

- Odometer is:
  - Relatively accurate model of wheel turn
  - Very inaccurate model of actual movement
- Actual position = odometer X,Y,θ + random noise



Simulation Implementation

- Start with odometer readings
- Add linear correction factor
  - $X = a_x * X + b_x$
  - $Y = a_y * Y + b_y$
  - $\theta = a_\theta * \theta + b_\theta$

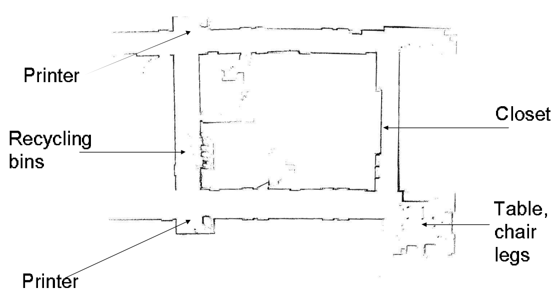
  Linear correction (determined experimentally)
- Add noise from the normal distribution
  - $X = X + N(0, s_x)$
  - $Y = Y + N(0, s_x)$
  - $\theta = \theta + N(0, s_\theta)$

  $N(\mu, s)$ returns random noise from normal distribution with mean μ and standard deviation s (standard deviation determined experimentally)

Main Loop

- Sample n robot states
- For each state
  - Simulate next state (action model)
  - Weight states (observation model)
  - Normalize
- Repeat

Internal Map Representation

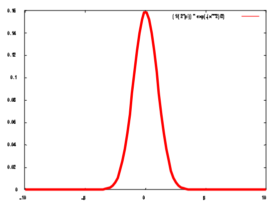


LSRC Second Floor

Laser Error Model

- Laser measures distance at 180 one degree increments in front of the robot (height is fixed)
- Laser rangefinder errors also have a normal distribution

Prob. of measurement



Distance from closest occupied square to endpoint of laser cast

Laser Error Model Contd.

- Probability of error in measurement k for sample i (normal)

$$p_{ik}(x_k) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-x_k^2}{2\sigma^2}}$$

- x_k is distance of laser endpoint to closest obstacle
- σ is standard deviation in this measurement (estimated experimentally), usually a few cm.

Laser Error Model Contd.

- Laser measurements are independent
- Weight of sample is product of errors:

$$p_i = \prod_k p_{ik}$$

- Note: Good to bound x to prevent a single bad measurement from making p_i too small

- Compute new weights for all particles:
- for i=1 to n
  - samples[i].p = p_i

Main Loop

- Sample n robot states
- For each state
  - Simulate next state (action model)
  - Weight states (observation model)
  - Normalize
- Repeat

Normalize Weights

- Sum of weights should be 1.0
- total = 0.0
- for i=1 to n
  - total = total + samples[i].p
- for i=1 to n
  - samples[i].p = samples[i].p/total

Main Loop

- Sample n robot states
- For each state
  - Simulate next state (action model)
  - Weight states (observation model)
  - Normalize
- Repeat

How do we use this?

Best Guess of Position

- Recover best guess of true position from weighted average of particle positions:

$$\bar{x} = \sum_i sample[i].x * sample[i].p$$